Sto2Auc: A Stochastic Optimal Bidding Strategy for Microgrids

Dou An, Qingyu Yang, Wei Yu, Xinyu Yang, Xinwen Fu and Wei Zhao

Abstract—Microgrids (MGs) have attracted growing attention due to self-sufficiency and self-healing properties. Nonetheless, the intermittent nature and uncertainty of distributed energy resources and load demands remain challenging issues in balancing demands and managing energy resources in MGs. Existing research efforts mainly focus on developing techniques to enable interactions between local MGs and the utility grid, which leads to high line power losses and operation costs. In this paper, we present the Sto2Auc framework to address the issue of stochastic optimal bidding problem for a system with MGs. First, the optimal bidding problem is formulated as a two-stage stochastic programming process, which aims to minimize the system operation cost and obtain optimal energy capacity of MGs by the microgrid controller (MGCC). Uncertainties arise from both energy supply and demand, which are considered in the decision-making process, and random parameters representing those uncertainties are captured by using the Monte Carlo method. Second, to enable optimal electricity trading between the insufficient and surplus MGs, we propose a distributed double auction (DDA)-based scheme, which is proven to converge to the optimal social welfare of the system with MGs, while the implemented DDA scheme achieves good performance with respect to social welfare, demand insufficiency, and MGCC profit.

Keywords—Microgrids, optimal bidding, uncertainties, stochastic programming, double auction, Internet of Things applications.

This work was supported in part by The National Science Foundation of China under Grant 61673315, 61573115, 61402356, 61572398, in part by the Fundamental Research Funds for the Central Universities under Grant xkj:2015010, in part by the State Key Laboratory of Manufacturing and System Engineering, in part by the Shaanxi International Cooperation and Exchange Program under Grant 2017KW-039, in part by the Science and Technology Fund of Macau Project under Grant 061/2011/A3 and Grant 092/2014/A2, in part by the University of Macau Project under Grant MYRG112-FST12-ZW and Grant MYRG2015-00165-FST, in part by the University System of Maryland Fund. (Corresponding authors: Qingyu Yang; Wei Yu)

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I. INTRODUCTION

The smart grid that is a typical Internet of Things (IoT) application [1], [2], [3], [4], [5], [6], [7], [8] and its goal is to make the process of electricity distribution reliable, economical, and sustainable [9], [10]. Microgrids (MGs), as one of the key elements in the distribution side of the smart grid, have received growing attention because of their salient features (e.g., environmental friendliness, reliability, self-sustainability, and social benefits). MGs in the smart grid are commonly operated in an island, connected to the utility grid, and are integrated with distributed energy resources, loads, batteries, and other electric components [11], [12].

Fig. 1 illustrates a typical grid-connected system with MGs. Through being connected to the utility grid, denoted as “grid-connected mode”, MGs can sell extra power to the utility grid and buy power from it whenever necessary. The Microgrid Center Controller (MGCC), also acting as an aggregator, aims at minimizing the operation cost of MGs while satisfying market efficiency and responsibilities to individual participants. Thus, how to efficiently manage energy resources in a system with MGs is a critical challenge. A number of research efforts have been devoted to address this issue [13], [12], [14].

In a system with MGs, variabilities and uncertainties raised by renewable energy resources and load demands make energy resource management challenging. To deal with this issue, stochastic energy management schemes have been developed. For example, Su et al. in [15] presented a stochastic method for the scheduling of energy resources in MGs, including a variety of renewable energy resources. In this direction, bidding strategies for MGs have been studied as well. For example, Nguyen et al. in [16] investigated a joint energy bidding and scheduling problem in a MG. In their study, uncertainties of renewable energy resources are compensated for by thermal dynamic characteristics of buildings.

Nonetheless, existing research efforts on energy resource management in systems with MGs primarily focus on interactions between MGs and the utility grid. To meet demands, the MGs will generate power via renewable energy resources. Nonetheless, those renewable energy resources cannot guarantee the stability of power generation due to their intermittent and uncertain nature. Because of the varying numbers of renewable energy resources in different MGs, the amount of energy generated locally in an MG can be larger or smaller than the local demand. Thus, as the supply of an MG is often not equal to demand, the MG needs to sell to or buy power from a primary power generation source (e.g., utility grid) [16]. Existing efforts typically focus on selling the surplus energy from MGs to the utility grid at a low price through the
MGCC. The utility grid then delivers the energy to insufficient MGs in the real-time market at a higher price [17]. In this way, benefits to MGs will be only limited. In addition, transmitting power from MGs (which commonly operates at low voltages) to the utility grid (which commonly operates at high voltages) will incur high transmission costs and line losses.

Thus, it will be more efficient for MGs to trade surplus energy to both the utility grid and other nearby MGs directly. How to develop techniques to enable efficient trading between MGs and the utility grid, as well as trading among local MGs directly, remain open issues. In addition, uncertainties from both demand and supply sides could affect the efficiency of system operation. To address the aforementioned issues, in this paper we propose the Sto2Auc framework, in which the optimal energy capacity of each individual MG is obtained by solving a two-stage stochastic programming problem. We consider local MGs as energy suppliers when locally generated energy is in excess, and as energy consumers when locally generated energy is insufficient. Under the framework, we also propose a secondary market to enable trading among MGs, and develop a distributed double auction (DDA) scheme to provide optimal energy bidding in the secondary market.

The main contributions of this paper can be summarized as follows:

First, we present a stochastic model for optimizing the operation of MGs, which aims to minimize the operation cost of the system. To address uncertainties from both renewable energy resources and demands, the optimal bidding problem is formalized as a two-stage stochastic programming process, in which uncertainty factors (e.g., renewable energy resources, load demand, and electricity prices) are captured through Monte Carlo-based methods. By solving the stochastic optimization model, the optimal energy capacity of each MG can be obtained by the MGCC.

Second, we propose to develop a secondary electricity market, in which MGs are treated in the same manner as other market entities. The market will be used when there exist surplus and insufficient MGs simultaneously, and energy trading among geographically close MGs is allowed. In the conference version of this paper [18], we have investigated a centralized benchmark Dynamic Backtrack Energy Trading (DBET) scheme to allocate surplus energy in the system with MGs, the schedule of power delivery from surplus MGs to insufficient MGs is based on the distance of power transmission for the sake of energy delivery cost. The clearing price in the secondary market is determined by the Cournot equilibrium.

Third, we present a distributed double auction (DDA) incentive scheme, which enables efficient energy bidding among local MGs in the secondary market. In this scheme, we take the social welfare of all MGs into consideration, and the MGs participate the auction scheme according to their own willingness, in which insufficient and surplus MGs are considered as buyers and sellers through their agents, respectively. Particularly, we first formalize the double auction problem as a Social Welfare Maximization (SWM) problem [19], which aims to maximize the utility of insufficient MGs (buyers) and surplus MGs (sellers). Then, the SWM problem is mapped into an Optimal Allocation Problem (OAP), which seeks to obtain optimal payment and allocation rules of the auction scheme. The optimal payment rules can be further determined by substituting Karush-Kuhn-Tucker conditions into the SWM and OAP problems. In addition, the optimal allocation rules of insufficient and surplus MGs are designed by solving their sub-problems, respectively. To solve the above optimization problems efficiently, a distributed double auction (DDA) scheme is proposed. In DDA, the Lagrange variables are updated via sub-gradient mechanism. Through theoretical analysis, we prove that our DDA scheme is capable of converging to the optimal social welfare, and satisfying the properties of individual rationality and (weak) budget balance.

Fourth, we conduct extensive experiments on a modified IEEE-33 bus-based system that consists of a number of MGs. Our experimental results show that, when our proposed Sto2Auc framework is used, the operation cost of the system will be reduced and the utility of the individual MG will be enhanced, the DDA scheme achieves better performance than the DBET scheme. In particular, the operation cost of the system with MGs is reduced by 10% when the DDA scheme is in place. In addition, our proposed DDA scheme, as part of the framework, is able to converge to optimal social welfare after a finite number of iterations, and further achieves good performance with respect to demand insufficiency and the MGCC’s profit.

This paper is an extension of our prior work in [18]. In addition to the original content presented in the conference version, substantial material additions have been made to this extended journal version, including considerable descriptions of the Sto2Auc framework, the supplementary modeling of the double auction scheme and the optimal solution of surplus energy allocation, an additional performance evaluation of the distributed double auction scheme, and extensive review of related research efforts. The remainder of this paper is organized as follows. In Section II, we present the overview of our approach, including the basic idea of our designed framework and the market model we considered in the paper. In Section III, we introduce the optimal bidding problem formalization, including the objective function and the constraints. In Section IV, we describe the distributed double auction (DDA) scheme, which enables optimal energy bidding among local
MGs. In Section V, we present our performance evaluation and results. We conduct literature reviews in Section VI. Finally, we conclude the paper in Section VII.

II. Sto2Auc Framework and Electricity Market Model

In this section, we first present the Sto2Auc framework and then introduce the electricity market model.

A. Sto2Auc Framework

In this paper, we consider a smart grid system that consists of multiple MGs that operate in a grid-connected mode. In this system, the capacity of selling surplus electricity or buying from an electricity market is allowed. Fig. 2 illustrates the system model. As we can see from the figure, the system is composed of a cluster of MGs, the agents of the MGs, and the MGCC. In this system, local MGs consist of several renewable energy generation units, conventional energy generation units, a number of residential consumers with load demands, and a battery storage facility. The agent is responsible for collecting energy usage information and interacting with the MGCC. The renewable energy generation units include solar panels and wind turbines. In this study, conventional generation units are referred to as non-renewable generation units (micro-turbines, fuel cells, diesel generators, etc.). We assume that electricity can be transported among MGs, especially MGs that are geographically close to each other. Notice that transmission techniques among MGs is outside of the scope of this paper.

Generally speaking, the MGCC tends to minimize the cost of a system of MGs by collecting information such as energy generation and demand of local MGs, energy delivery among local MGs, energy delivery between MGs and the utility grid, including day-ahead bid submissions, real-time electricity purchasing and selling, and others. The locally insufficient and surplus amounts of energy required to balance load demands can be covered by interacting with the utility grid [20], or with other MGs. The MGCC can make decisions on purchasing electricity from the market or selling electricity to the market based on the status of local units, as well as other factors (the electricity price, states of conventional units, operation cost, etc.).

Nonetheless, variabilities and uncertainties are raised by renewable energy resources, and load demands challenge energy resource management in the system with MGs. On the supply side, wind and solar power output are highly uncertain and unpredictable. Even a small error in the prediction can result in great errors in real-time operations. On the demand side, factors (natural disasters, plug-in vehicles, personal habits of energy use, weather and temperature, etc.) make it difficult to accurately predict energy usage. The effectiveness of integrating MGs can be affected by those uncertainties.

In addition, to meet the growing demand of electricity, and to enhance efficient and reliable energy service, modern power generation techniques (micro-turbines, renewable energy, clean and efficient fossil fuels, distributed generations, etc.) have been used in the system with MGs. The increasing local power generation can lead to surplus electricity for MGs in a particular time duration, and the excess has to be traded and used due to the limited capacity of local storage. Due to the low voltage property of MGs, transmitting power from MGs to the utility grid can pose high transmission costs and line losses. Thus, it is important to develop techniques that allow efficient trading between MGs directly, which has not been addressed in the past. Through the trading process, the surplus electricity can be sold to insufficient MGs based on the market clearing price settled by the MGCC.

In this paper, we propose a Sto2Auc framework, as shown in Fig. 3, to address the issue of the efficient trading problem, which considers energy interactions among MGs locally, as well as uncertainties raised by renewable energy resources and varying demand in MGs. As we can see from Fig. 3, the proposed Sto2Auc framework is composed of two core components. The component on the left indicates the two-stage Stochastic Programming mechanism that aims to minimize the operation cost of the multi-MG system and leads to the optimal energy capacity of the MGs. The component on the right of the figure indicates the distributed double auction that enables energy trading among local MGs optimally.

In the designed Sto2Auc framework, the trading problem in the system is formalized as a two-stage stochastic programming problem, aimed at minimizing the cost of the multi-MG system operation. In the problem formalization, we consider parameters that capture different uncertainties, including the randomness of weather forecasting, load demand, and electricity price. We use the Monte Carlo-based method to construct scenarios based on the distribution functions of those parameters to capture uncertainties. By solving the two-stage stochastic programming process, the optimal energy capacity for individual MGs can be obtained.

We also consider the secondary market model to enable effective trading among entities. In the intended market, energy transactions among local MGs are allowed, meaning that surplus MGs are able to transmit electricity directly to insufficient MGs. To enable energy trading among local MGs, we first present a centralized Cournot Equilibrium-based scheme as a baseline scheme. We then design a distributed double auction (DDA) scheme, shown as the rightmost portion of Fig. 3, in which the social welfare of all MGs is taken into consideration, and a distributed double auction algorithm is proposed to enable optimal trading between surplus MGs (sellers) and insufficient MGs (buyers). As is shown in Fig. 3, we first
formalize the double auction problem as a social welfare maximization (SWM) problem, which can be further mapped into an optimal allocation problem (OAP), which is illustrated as step 1 in the figure.

Then, the optimal energy allocation rules of the insufficient and surplus MGs are obtained via the agents solving their sub-problems: insufficient MG’s optimal problem (IMO) and surplus MG’s optimal problem (SMO), iteratively, which are illustrated as step 2 to step 5 in the figure. Finally, the insufficient MGs optimal payment rule and the surplus MGs optimal income rule in the double auction scheme can be determined by comparing KKT conditions of SWM and OAP problems, and the optimal conditions of IMO and SMO problems, which are illustrated as step 6 to step 8 in the figure. The detailed procedure of the distributed double auction scheme can be referred as Algorithm 1 in Section IV-C.

B. Market Model

The electricity market that we consider is a deregulated market [21]. As we can see from Fig. 2, the electricity market is divided into two parts: the primary market and the secondary market. The primary market is assumed to be a traditional energy market, which consists of energy trading between MGs and the utility grid through the coordination of the MGCC. In this market, trading prices are day-ahead or real-time electricity prices. In each day, agents of local MGs need to forecast demands and the amount of renewable energy resources generated, and then submit hourly bids to the day-ahead market before real-time delivery. Bids from the agent include selling and buying electricity bids, denoted as tuples \( \text{\{amount, price\}} \). Meanwhile, to replenish the energy gap or clear the surplus electricity, the MGs are also allowed to participate in the real-time market. The market clearing price will be computed by the system operator at the MGCC after the demand and supply information is collected through the market.

Recall that agents usually provide selling bids at a relatively low price, which is usually far below the generation cost of local units [22], while offering buying bids at high prices to guarantee that bids are always admissible. Also, because of the low voltage characteristics of MGs, a relatively frequent power transmission to the utility grid will incur line losses, posing an increased operational cost. For this purpose, to guarantee energy interactions among local MGs, we define the secondary market as a supplement of the primary market. Notice that from the view of electricity companies and market operators, the secondary market should be limited and the amount of electricity distributed among MGs should not be very high.

We assume that there must exist some MGs that want to purchase power, while others want to sell power, in a time window. The MGs that want to purchase power are denoted as insufficient MGs, while the MGs that want to sell power are denoted as surplus MGs. When there are both insufficient MGs and surplus MGs in the system, the secondary market can be used to enable trading between surplus and insufficient MGs. The trading process can be determined by the MGCC based on our proposed double auction scheme in Section IV. We also assume that the involved MGs in the secondary market are those that are geographically close for the sake of reducing transmission cost and line losses. It is worth noting that the secondary market is useful to enable efficient energy distribution and to reduce operation costs in a system with

![Fig. 3: Sto2Auc framework](image-url)

### Table I: Notations

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M, N, K, T, S )</td>
<td>Number of local units, MGs, renewable energy resources, time slots, and scenarios</td>
</tr>
<tr>
<td>( \delta_{i,t} )</td>
<td>Penalty factor of deviation between day-ahead bids and real-time demand</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Transmission cost factor related distance between MGs</td>
</tr>
<tr>
<td>( \gamma_{cha}, \gamma_{dis} )</td>
<td>Charging/discharging efficiency of battery</td>
</tr>
<tr>
<td>( \omega_{cha}, \omega_{dis} )</td>
<td>Battery charging/discharging degradation</td>
</tr>
<tr>
<td>( \Delta_T )</td>
<td>Duration of time slot (h)</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Probability of Monte-Carlo scenario s</td>
</tr>
<tr>
<td>( \nu_{da, rt} )</td>
<td>Day-ahead and real-time electricity price in primary market ($/kWh)</td>
</tr>
<tr>
<td>( CG )</td>
<td>Generation cost of local units ($/kWh)</td>
</tr>
<tr>
<td>( pm_i^m )</td>
<td>Power generation of local unit ( m ) (kW)</td>
</tr>
<tr>
<td>( St_{i,m}, SP_{i,m} )</td>
<td>Start-up and shutdown cost of unit ( m ) ($)</td>
</tr>
<tr>
<td>( \nu_{b, t} )</td>
<td>Scheduled bids and real-time delivery (kW)</td>
</tr>
<tr>
<td>( p_{cha}, p_{dis} )</td>
<td>Purchasing and selling power from MGCC and other MG (kW)</td>
</tr>
<tr>
<td>( pm_{m}, P_{mae} )</td>
<td>Selling power to MGCC and other MG (kW)</td>
</tr>
<tr>
<td>( \phi_{i,m} )</td>
<td>Generation of renewable energy resource ( k ) (kW)</td>
</tr>
<tr>
<td>( \nu_{cha, dis} )</td>
<td>Charging/discharging power of battery (kW)</td>
</tr>
<tr>
<td>( pm_{min}, P_{max} )</td>
<td>Minimum/Maximum power generation of unit(kW)</td>
</tr>
<tr>
<td>( \phi_{grid} )</td>
<td>Shutoff/startup offer cost of unit ($)</td>
</tr>
<tr>
<td>( \phi_{dis}, E_{max} )</td>
<td>Maximum power of battery ( i ) charging/discharging unit(kW)</td>
</tr>
<tr>
<td>( \phi_{cap}, E_{max} )</td>
<td>Capacity of battery at time slot ( t ) (kWh)</td>
</tr>
<tr>
<td>( \phi_{b, t} )</td>
<td>Lower/Upper bound of battery capacity (kWh)</td>
</tr>
<tr>
<td>( \phi_{mg}, E_{max} )</td>
<td>Lower/Upper bound of MG interaction with main grid (kWh)</td>
</tr>
<tr>
<td>( \phi_{status} )</td>
<td>Status of local units at time slot ( t )</td>
</tr>
<tr>
<td>( \lambda, \lambda' )</td>
<td>Binary variables 0, 1 that identify whether the bid is successful or not</td>
</tr>
<tr>
<td>( q_i, q_i' )</td>
<td>Lagrangian vector in the double auction</td>
</tr>
<tr>
<td>( U_i(q_i'), C_j(q_j') )</td>
<td>Demand and supply vector of MGs</td>
</tr>
<tr>
<td>( P_i(b_i'), P_j(s_j') )</td>
<td>Utilities of insufficient and surplus MGs</td>
</tr>
<tr>
<td>( \nu_{i,t} )</td>
<td>Payment and income rules</td>
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</table>
MGs. To this end, we design a distributed double auction scheme to enable energy trading among MGs in Section IV. The detailed description of the notations used in this paper can be found in Table I.

III. TWO-STAGE STOCHASTIC OPTIMIZATION MODEL

In this section, we present the stochastic optimization model and the corresponding constraints.

A. Problem Formulation

We now present the two-stage stochastic programming problem [23] to minimize the operation cost of the system with MGs. Generally speaking, the stochastic programming process is a method that aims to minimize the cost in a number of scenarios constructed by a Monte Carlo-based scheme, while being obligated to uncertainties in the problem. The basic idea of the two-stage programming process is to conduct a recursion process to make corrective decisions after a random event occurs.

In our two-stage stochastic programming process, the input and output of the problem are parameters used to capture uncertainties and decisions for both the first and second stage. At the beginning, the initial decision on day-ahead energy bids should be made in the first stage. Then, uncertainty factors from renewable energy resources (wind, solar, etc.), load demand, ambient temperature, and electricity price information are mimicked based on scenarios constructed via Monte Carlo-based method, which affects the outcome of the first-stage decision. Decisions will then be made at the second stage to compensate for uncertainties. The optimal decision in the first stage has an objective of identifying the amount of optimal power to be purchased or sold, and the commitment of distributed energy generation units over the next 24 hours. Decisions in the second stage consist of the power dispatch of all local generating units, the amount of electricity purchased or sold, and decisions for battery charging and discharging. Notice that it is a common practice that the decisions made in the first stage do not vary across scenarios constructed by Monte Carlo-based scheme in the second stage.

We now describe the stochastic problem. The objective function is described as follows:

\[
\min \sum_{N} \sum_{M} \sum_{T} (C^{G} P_{i,t}^{k} + \sum_{t} \sum_{j} (SU_{i,t} + SD_{i,t})) + \sum_{s} \sum_{t} \{ P_{sch}^{r_{da}} + I(P_{i,t}^{sch} < P_{i,t}^{DA}) \},
\]

\[
(P_{B, MGCC}^{r_{rt}} + P_{B, MG}^{r_{MG}}) - I(P_{sch}^{r_{rt}} > P_{i,t}^{DA}),
\]

\[
(P_{S, MGCC}^{r_{rt}} + P_{S, MG}^{r_{MG}}) - Cost(battery) - \delta_{i,t} \{ P_{sch}^{D} - P_{sch}^{s} - P_{sch}^{k} \}. (1)
\]

Recall that in the system shown in Fig. 2, the operation cost in each MG consists of generation, startup and shutdown cost of local units, energy purchasing cost, energy selling revenue, battery discharging and charging cost, the penalty cost for considering the deviation between the day-ahead scheduling and real-time delivery, as well as energy transmission loss.

We now explain the objective function in detail. The first and second parts \((C^{G} P_{i,t}^{k})\) of the objective function 1 correspond to the startup and shutdown cost, and the generation cost associated with local generators, respectively. The generation cost consists of the cost for both local conventional generators and renewable energy generators. In our model, renewable energy resources are wind and solar. Thus, corresponding uncertainties are wind speed and ambient temperature.

The third, fourth, and fifth parts \((P_{Sch}^{r_{da}}, P_{sch}^{r_{rt}})\) in the objective function 1 are associated with the cost of day-ahead bids, the amount of electricity purchased and sold with the utility grid and among local MGs, respectively. Here, \(P_{sch}^{r_{da}}\) is referred to as the amount of energy purchased or sold for day-ahead bids. \(I(.)\) is referred to as an indicator function, in which positive parts represent the amount of electricity purchased from the utility grid or surplus MGs, and negative parts represent the amount of electricity sold to the utility grid or insufficient MGs. Here, \(Cost_{i,t}^{B, MG}\) and \(Rev_{i,t}^{S, MG}\) are the cost and revenue for energy trading associated with individual MGs.

The last three parts \((Cost(battery), \delta_{i,t} \{ P_{i,t}^{DA} - P_{sch}^{s} \}, P_{sch}^{s})\) in the objective function 1 are battery charging and discharging costs, the penalty cost for the deviation between day-ahead schedule and real time delivery, and the transmission cost between two local MGs. Here, \(\delta_{i,t}\) is referred to as the penalty factor for electricity transaction deviation. Recall that due to the aforementioned uncertainties in the system, it is possible to over-commit on the day-ahead schedule. Nonetheless, the penalty factor is to reduce the difference between real time electricity delivery and day-ahead scheduling through minimizing over-commitment. In this way, fluctuations in power transmission can be avoided.

B. Constraints

In the optimization problem, we need to consider various constraints. The first constraint is to balance power. For each MG in the system, the following power balance constraint needs to be satisfied:

\[
\sum_{m} P_{i,t}^{m} + \sum_{k} P_{i,t}^{k} + \sum_{j} P_{j,t}^{dis} = \sum_{j} P_{j,t}^{cha} + \sum_{j} P_{j,t}^{D}, (2)
\]

\[
\sum_{m} P_{i,t}^{B, MGCC} + \sum_{t} P_{i,t}^{B, MG} - \sum_{m} P_{i,t}^{S, MGCC} - \sum_{m} P_{i,t}^{S, MG},
\]

\[
= P_{i,t}^{DA} - P_{sch}^{s}. (3)
\]

In any time duration, the power balance constraint should be satisfied. This means that the sum of the total power generation from all local generation units and the discharging power from battery units, and the total power sold to the utility grid and other MGs, must be equal to the sum of the amount of purchased power from the utility grid and other MGs, and amount of battery charged. In Equation (3), the left side is the amount of electricity that MGs trade in the real-time market,
which is equal to the difference between day-ahead scheduling and real-time delivery in the right side.

The second constraint is related to conventional unit constraints. The operating cost of conventional units can be modeled approximately by,

$$ C^G P_{i,t} = a_i + b_i P_{i,t} + c_i P_{i,t}^2, \quad (4) $$

where $P_{i,t}$ is output power, $a_i$, $b_i$, and $c_i$ are generation cost factors of local conventional generation unit $i$. The output power generation should also be limited by

$$ P_{\text{min}}^i \leq P_{i,t} \leq P_{\text{max}}^i. \quad (5) $$

The third constraint is associated with start-up and shutdown costs, which are listed as follows:

$$ SU_{i,t} \geq CU_{i,t} (I_{i,t} - I_{i,t-1}); \quad SU_{i,t} \geq 0, \quad (6) $$

$$ SD_{i,t} \geq CD_{i,t} (I_{i,t-1} - I_{i,t}); \quad SD_{i,t} \geq 0, $$

where $I$ is the indicator function with a value of 0 or 1, and $CU$ and $CD$ are the cost for the startup and shutdown of conventional generation unit.

The fourth constraint is related to battery. The capacity of storage in each MG should satisfy the following constraint:

$$ 0 \leq P_{i,t}^{\text{dis}} \leq U_{i,t} u_s P_{i,t}^\max, $$

$$ 0 \leq P_{i,t}^{\text{cha}} \leq (1 - U_{i,t}) u_s P_{i,t}^\max, $$

$$ E_{i,t+1} = E_{i,t} + \gamma_{\text{cha}} P_{i,t}^{\text{cha}} \Delta T - \frac{P_{i,t}^{\text{dis}} \Delta T}{\gamma_{\text{dis}}}, \quad (7) $$

$$ E_{i,t} \leq E_{i,t} \leq E_{i,t}^\max. $$

For the battery in each MG $i$, the above constraint considers charging and discharging limits, the battery state, and the upper and lower bounds of battery capacity. Here, $U_{i,t}$ is an indicator function with a value of 0 or 1, representing the battery is either in a charging or discharging state, respectively. One purpose of using the charging and discharging states is to ensure that charging and discharging processes are performed simultaneously. Also, $\gamma_{\text{cha}}$ and $\gamma_{\text{dis}}$ are referred to as the efficiency of the charging and discharging processes, respectively.

In the objective function 1, the degradation cost of battery charging and discharging is also denoted as $\text{Cost(battery)}$, which can be derived by

$$ \text{Cost(battery)} = \gamma_{\text{cha}} \omega_{\text{cha}} P_{i,t}^{\text{cha}} \Delta T - \frac{P_{i,t}^{\text{dis}} \Delta T \omega_{\text{dis}}}{\gamma_{\text{dis}}}. \quad (8) $$

The next constraint is related to the energy output from renewable energy resources. Based on our prior work [24], we can derive the upper and lower bound [25] according to the probabilistic density function of the distribution of energy output from the renewable energy resources. Then, the upper and lower bound of energy output from renewable energy generation is as follows:

$$ P_{\text{res}}^\min \leq P_{i,t}^\text{res} \leq P_{\text{res}}^\max. \quad (9) $$

Finally, the maximum capacity of electricity interactions between MGs and the utility grid cannot exceed the capacity limit of physical transmission line. We thus have

$$ P_{\text{grid}}^\min \leq P_{i,t}^\text{grid} \leq P_{\text{grid}}^\max. \quad (10) $$

Recall that the third through fifth parts of the objective function in Equation (1) represent the amount of electricity purchased and sold in the system, including the amount of energy traded among local MGs, and between MGs and the utility grid. The energy trading between MGs and the utility grid will be performed in the primary market, which is composed of day-ahead and real time markets. Trading prices are obtained through prediction. To design effective electricity interaction among MGs, we introduce the Cournot Equilibrium [26]-based DBET scheme in the secondary market. The detail of the design algorithm can be found in [18].

IV. DISTRIBUTED DOUBLE AUCTION (DDA) SCHEME

After obtaining the real time energy capacity of each individual MG by solving the two-stage stochastic programming problem presented in Section III, we present a distributed double auction (DDA) scheme, in which, through the agent of MG, potential buyers submit their bids with amounts and prices of energy, and sellers submit their requests with amounts and prices to the MGCC. Then, a price will be determined by the MGCC to clear the market. By doing so, the secondary market is bilateral and competitive, where social welfare of all MGs in the system can be maximized.

A double auction scheme is composed of allocation rule and pricing rule, in which the allocation rule indicates the energy transactions among local MGs, while the pricing rule indicates payment and income of the buyers and the sellers. The goal of the distributed double auction scheme is to design rational allocation and pricing rules to achieve maximum social welfare. In the following, we first present the problem formalization, and then present the distributed double auction scheme to solve this problem in detail.

A. Social Welfare Maximization

Considering that in time duration $t$, there are $B_N$ insufficient MGs in the system, referred as the buyers set $B = \{i | i = 1, 2, \ldots, B_N\}$. Accordingly, there are $S_N$ surplus MGs in the system, referred as the sellers set $S = \{j | j = 1, 2, \ldots, S_N\}$. Meanwhile, the maximum demand and the available electricity are $b_i$ and $s_j$, respectively. In this paper, we assume that the amount of excess electricity of all surplus MGs is less than the sum of all the electricity needed by insufficient MGs, and can be sold out in the auction process. This means that the buying amount is greater than the selling amount in double auction strategy. The remaining unfulfilled electricity will be provided by the utility grid in the primary market.

In each time slot, each insufficient MG $i \in B$ is assumed to purchase $q_{i,j}^b$ amount of electricity from surplus MGs $j \in S$. We denote the insufficient MGs purchasing vector from all the surplus MGs as $q_i^b = (q_{i,j}^b; \forall j \in S)$. Also, we define the utility of $i^{th}$ buyer when purchasing $q_i^b$ amount of electricity to all surplus MGs in $S$ as $U_i(q_i^b)$. The utility is assumed to be positive, increasing and concave with $q_i^b$. With respect to sellers, we denote $q_{j,i}^s$ as the amount that $j^{th}$ surplus MGs supply to the insufficient MG $i \in B$. Similarly, the supply vector of surplus MGs to all insufficient MGs as $q_j^s = (q_{j,i}^s; \forall i \in B)$. We denote $C_j(q_j^s)$ as an increasing,
positive and strictly convex function of \( q_i^b \), which indicates the cost raised by energy delivery to insufficient MG \( i \).

Due to the selfishness of insufficient and surplus MGs’ agents, the objectives of MG agents are inconsistent. In other words, the insufficient MGs’ agents aim to purchase electricity with as low a payment as possible, while the surplus MGs’ agents seek to deliver electricity at the lowest transmission cost. As the ‘central nervous’ of the MG system, the MGCC is responsible for enabling the system to achieve balance by taking both insufficient and surplus MGs utilities into consideration, as well as solving the following social welfare maximization (SWM) problem.

**Definition 1 (Social welfare maximization).** The social welfare maximization is to maximize the difference between the sum of winning insufficient MGs valuations and winning surplus MGs costs [27]. The objective and constraints are expressed as follows:

\[
\max_{q^b, q^s} \sum_{i=1}^{B_N} U_i^b(q^b_i) - \sum_{j=1}^{S_N} C_j(q^s_j) \quad \text{(11)}
\]

\[
\text{S.t.} \quad \sum_{j=1}^{S_N} q^s_{j,i} \leq b_i, \forall i \in B; \quad \text{(12)}
\]

\[
\sum_{i=1}^{B_N} q^b_{i,j} \leq s_j, \forall j \in S; \quad \text{(13)}
\]

\[
q^b_{i,j} = q^s_{j,i}, \forall i \in B, j \in S; \quad \text{(14)}
\]

\[
q^b_{i,j} \geq 0, q^s_{j,i} \geq 0, \forall i \in B, j \in S. \quad \text{(15)}
\]

The objective function, described in Equation (11), is denoted as the Social Welfare, which refers to the difference between the sum of winning buyers valuations and the sum of winning sellers costs. We further explain the constraints in detail. The first constraint, described by Equation (12), indicates that the amount of energy purchased by insufficient MGs is limited by \( b_i \), in the secondary market; the second constraint, Equation 13, denotes that the amount of energy supply from surplus MGs is limited by \( s_j \). The third constraint, the result of Equation 14, ensures power balance, and the fourth constraint, Equation 15 indicates that there exist non-negative transactions of the MGs in the secondary market. In addition, based on the definitions of buyer and seller utilities, we can obtain that the objective function is strictly concave. Furthermore, the constraints are convex and compact. Thus, the optimal solution of SWM can be derived by KKT conditions. We first relax the constraints in Equation (11) to obtain the Lagrange function, and then derive the optimal solution by

\[
L(\lambda, \mu, \nu, q^b, q^s) = \sum_{i=1}^{B_N} U_i^b(q^b_i) - \sum_{j=1}^{S_N} C_j(q^s_j),
\]

\[
-\sum_{i=1}^{B_N} \lambda^b_i \left( \sum_{j=1}^{S_N} q^b_{i,j} - b_i \right) - \sum_{j=1}^{S_N} \mu^s_j \left( \sum_{i=1}^{B_N} q^s_{j,i} - s_j \right),
\]

\[
+\sum_{i=1}^{B_N} \sum_{j=1}^{S_N} \nu^b_{i,j} (q^b_{i,j} - q^s_{j,i}). \quad \text{(16)}
\]

Here, \( \lambda_i \in \mathbb{R}^+, \mu_j \in \mathbb{R}^+ \) and \( \nu_{i,j} \in \mathbb{R} \) are Lagrange multipliers, which correspond to the first, second and third constraints, respectively. The optimal values of Lagrange variables: \( \lambda^*_i, \mu^*_j, \nu^*_{i,j} \) and the transactions \( q_{i,j}^b, q_{i,j}^s \) can be derived from the following equations according to KKT conditions:

\[
\frac{\partial L(\lambda, \mu, \nu, q^b, q^s)}{\partial q^b_{i,j}} = 0 \Rightarrow \frac{\partial U_i^b(q^b_i)}{\partial q^b_{i,j}} = \lambda^*_i + \nu^*_{i,j}; \quad \text{(17)}
\]

\[
\frac{\partial L(\lambda, \mu, \nu, q^b, q^s)}{\partial q^s_{j,i}} = 0 \Rightarrow \frac{\partial U_j^s(q^s_j)}{\partial q^s_{j,i}} = \nu^*_{j,i} - \mu^*_j; \quad \text{(18)}
\]

\[
\lambda^*_i \left( \sum_{j=1}^{S_N} q^b_{i,j} - b_i \right) = 0, \quad \mu^*_j \left( \sum_{i=1}^{B_N} q^s_{j,i} - s_j \right) = 0; \quad \text{(19)}
\]

\[
\nu^*_{i,j} (q^b_{i,j} - q^s_{j,i}) = 0 \Rightarrow q^b_{i,j} = q^s_{j,i}; \quad \text{(20)}
\]

\[
q^b_{i,j}, q^s_{j,i}, \lambda_i, \mu_j \geq 0. \quad \text{(21)}
\]

**B. Desired Properties**

Notice that, in practice, it is infeasible for the MGCC to obtain the optimal solution for the social welfare maximization by solving the conditions from Equations (17) to (20). This is because some strategic agents could possibly misreport information (i.e., energy demands, energy supply or personal utilities). Thus, the objective for the MGCC is to design a feasible mechanism that induces the agents to report their truthful types, while maximizing the social welfare and ensuring the secondary market’s fairness and effectiveness. Such a double auction process shall satisfy the following economic properties: Individual Rationality, (Weak) Budget Balance and Strategy-Proofness. The detailed definitions of the properties are given as follows.

**Definition 2 (Individual Rationality).** A double auction scheme is considered individually rational when all participants should obtain non-negative utility, such that they have incentive to take part in the auction scheme.

**Definition 3 ((Weakly) Budget Balanced).** To ensure the auctioneer can obtain non-negative profit, the double auction scheme should satisfy the (weak) budget balance property, meaning that payments from the buyers to the MGCC should be equal or larger than those of the MGCC to sellers.

**Definition 4 (Strategy-Proof).** A double auction scheme is regarded as strategy-proof if, for any given MG \( i \in M \), the agent of MG achieves his/her maximum utilities if and only if he/she reports the truthful types, while misreporting types cannot improve his/her utilities. Strategy-proofness is also called truthful or dominant-strategy-incentive-compatible (DSIC) [28].

**C. Distributed Double Auction (DDA) Scheme**

We now introduce a distributed double auction (DDA) scheme for surplus energy allocation in the secondary electricity market, assuming that MG agents are price takers, which achieve social welfare maximization 11 while satisfying the desired economic properties outlined in Section IV-B.

The basic framework is that in each time slot, each insufficient MG \( i \in B \) submits bid \( b_i^b \) to surplus MG \( j \) for
per unit electricity, which is willing to purchase from MG $j$. Accordingly, the surplus MG $j \in S$ submits ask $a_{i,j}^s$ for per unit electricity that it is selling to MG $i$. After collecting the bids and asks that reflect the current demand and supply from the MGs’ agents, the MGCC will determine the allocation rule $(q^b, q^s)$ to assign the surplus electricity for buyers and sellers by addressing the following optimal allocation problem (OAP) inspired by the allocation rule in [29]:

$\text{OAP: } \max \sum_{i=1}^{B_N} \sum_{j=1}^{S_N} (b_{i,j} \log q_{i,j}^b - \frac{1}{2} s_{i,j}^s (q_{i,j}^s)^2).$ \hspace{1cm} (22)

Notice that OAP is subject to the same constraints as the SWM problem 11. Apparently, OAP has a strict concave objective function as well, and admits a unique optimal solution. The Lagrange relaxation of OAP can be expressed as:

$\tilde{L}(\lambda, \mu, \nu, q^b, q^s) = \sum_{i=1}^{B_N} \sum_{j=1}^{S_N} (b_{i,j} \log q_{i,j}^b - \frac{1}{2} s_{i,j}^s (q_{i,j}^s)^2),$

$- \sum_{i=1}^{B_N} \lambda_i \left( \sum_{j=1}^{S_N} q_{i,j}^b - b_i \right) - \sum_{i=1}^{B_N} \mu_j \left( \sum_{i=1}^{S_N} q_{i,j}^s - s_j \right)$

$+ \sum_{i=1}^{B_N} \sum_{j=1}^{S_N} \nu_{i,j} (q_{i,j}^b - q_{i,j}^s).$ \hspace{1cm} (23)

The corresponding KKT conditions yield the optimal Lagrange variables $\lambda_i^*, \mu_j^*, \nu_{i,j}^*$, and the transactions $q_{i,j}^{b*}, q_{i,j}^{s*}$ by solving the following equations:

$\partial \tilde{L}(\lambda, \mu, \nu, q^{b*}, q^{s*}) \bigg/ \partial q^b = 0 \Rightarrow \bar{q}_{i,j}^b = \frac{b_{i,j}}{\lambda_i^* + \nu_{i,j}^*};$ \hspace{1cm} (24)

$\partial \tilde{L}(\lambda, \mu, \nu, q^{b*}, q^{s*}) \bigg/ \partial q^s = 0 \Rightarrow \bar{q}_{i,j}^s = \frac{\bar{s}_{i,j}^s - \bar{\mu}_{i,j}^*}{\lambda_i^*};$ \hspace{1cm} (25)

$\lambda_i^* \left( \sum_{j=1}^{S_N} q_{i,j}^{b*} - b_i \right) = 0, \hspace{0.5cm} \mu_j^* \left( \sum_{i=1}^{B_N} q_{i,j}^{s*} - s_j \right) = 0;$ \hspace{1cm} (26)

$\bar{\nu}_{i,j}^* (q_{i,j}^{b*} - q_{i,j}^{s*}) = 0 \Rightarrow \bar{q}_{i,j}^{b*} = q_{i,j}^{s*};$ \hspace{1cm} (27)

$q_{i,j}^{b*}, q_{i,j}^{s*}, \lambda_i^*, \mu_j^* \geq 0.$ \hspace{1cm} (28)

We can observe that Equations 24 and 25 indicate the optimal electricity allocation rule for each insufficient and surplus MG, which are on the basis of bids and asks collected $b_{i,j}, s_{i,j}$ by MGCC and the Lagrange variables $\lambda_i^*, \nu_{i,j}^*$. To ensure that the optimal allocation rules are likewise suitable for the SWM problem 11, we match the KKT conditions in SWM and OAP, giving us:

$$\begin{cases}
    b_{i,j}^t = \frac{\partial U_i(q_{i,j}^{b*})}{\partial q_{i,j}^{b*}} \cdot q_{i,j}^{b*}, \\
    s_{i,j}^t = \frac{\partial I_j(q_{i,j}^{s*})}{\partial q_{i,j}^{s*}} \cdot \frac{1}{q_{i,j}^{s*}}.
\end{cases}$$ \hspace{1cm} (29)

This means that, when the surplus MG $j$ and insufficient MG $i$ submit their bids and asks according to Equation (29), the optimal solution obtained by MGCC in the SWM problem is equivalent to the optimal solution in OAP. Thus, the payment rule for the insufficient MGs and income rule for the surplus MGs that induce agents to submit bids and asks should be based on Equation (29).

We now denote $P_i(b_i^t)$ as the insufficient buyer $i \in B$’s payment to the auctioneer MGCC for receiving electricity $q_i^b$, while $I_j(s_j^t)$ as the surplus seller $j \in S$’s income from the MGCC for the supplied electricity. We can observe that the payment and income rules depend on the buyers’ asks and sellers’ bids. Due to the individual selfishness and rationality of MG agents, their optimal bids and asks are based on the willingness to maximize their own utility.

To this end, in each time slot, for a given optimal electricity allocation rule, defined in Equation (24), buyer $i$ is able to find its optimal bid $b_i^{t*}$ by solving the insufficient MG’s optimal (IMO) problem:

$\text{IMO: } \max_{b_{i,j}^t \geq 0} U_i(q_{i,j}^t(b_i^t)) - P_i(b_i^t).$ \hspace{1cm} (30)

Correspondingly, the surplus seller $j \in S$ is able to obtain its optimal asks $s_j^{t*}$ by solving the surplus MG’s optimal (SMO) problem:

$\text{SMO: } \max_{s_{j,i}^t > 0} I_j(q_{i,j}^t(s_j^t)) - C_j(s_j^t).$ \hspace{1cm} (31)

Notice that, because IMO and SMO are convex optimization problems, the optimal conditions of these problems can be expressed as:

$$\begin{cases}
    \partial U_i(q_{i,j}^t) \bigg/ \partial q_{i,j}^t = \frac{\partial P_i(b_i^t)}{\partial b_{i,j}^t}, \\
    \partial I_j(q_{i,j}^t) \bigg/ \partial q_{i,j}^t = \frac{\partial C_j(s_j^t)}{\partial s_{j,i}^t}.
\end{cases}$$ \hspace{1cm} (32)

These indicate the optimal responses of insufficient buyers and surplus sellers under optimal conditions of SWM and OAP. By substituting KKT conditions in Equations (17), (18), (24) and (25) to (32), the payment and income rules can be derived to induce the agents to submit according to Equation (29) as:

$$\begin{cases}
    b_{i,j}^t = \sum_{j=1}^{S_N} b_{i,j}, \\
    I_j(s_j^t) = \sum_{i=1}^{B_N} \frac{1}{s_{j,i}^t} \cdot (q_{i,j} - \mu_{i,j})^2.
\end{cases}$$ \hspace{1cm} (33)

At this point, the allocation rule, based on Equations (24) and (25), and the pricing rules, including the insufficient MGs payment rule and the surplus MGs income rule, are determined to induce the agents to submit their optimal bids and asks to ensure the maximum social welfare defined in Equation (11). In this case, the agents’ information (i.e., utility and cost of each MG) are fully known to the entities in the market. Nonetheless, because not all of this information is public to all entities in the market, the optimal social welfare cannot be achieved in one round.

To this end, a distributed algorithm that is executed iteratively is proposed to achieve the desired optimal social welfare. In this algorithm, the optimal energy capacity (either demand or supply) of each MG is derived according to the stochastic model defined in Equation (1). Thus, the sets of buyers and
sellers in the secondary market can be defined accordingly. Next, based on the scheme proposed in this section, the optimal bids of an insufficient MG can be obtained by the agent by solving the IMO problem defined in Equation (30). Meanwhile, the agent also obtains the optimal ask of a surplus MG by solving the SMO problem defined in Equation (31). After calculating the optimal bids and asks, the MG agents submit them to the MGCC for allocation and payment rule determination. Notice that the parallel process of optimal bid and ask computation enables efficient acceleration of the execution of the double auction scheme in achieving maximum social welfare.

Finally, by solving Equations (24) and (25) for the collected bids and asks, the MGCC determines and announces the allocation rules of the winning MG agents. Notice that the allocation rule is relevant to the variables \( S \) and \( B \), and the Lagrange variables. Moreover, the pricing rules obtained by solving insufficient and surplus MG problems IMO and SMO are related to the Lagrange variables. To this end, the above process can be executed iteratively by updating the Lagrange variables through a sub-gradient mechanism.

Algorithm 1 Distributed Double Auction Algorithm

Require: Optimal day-ahead scheduling bids and real-time demand for each MG \( m \) in time slot \( t: P_{m,t}^{s,c}, P_{m,t}^{D} \)
Ensure: \( x^*, y^*, \lambda, \nu \) and \( \mu \)

1: \( q \leftarrow 0; \)
2: Initialization: Energy matrix for each MG \( i: E_{m,t} = P_{m,t}^{s,c} - P_{m,t}^{D}, q_{i,0}^0, q_{i}^0, \mu_{i}^0, \nu_{i}^0 \); \)
3: repeat
4: \( \text{Set } B = \{ \text{Group of MGs buying energy } | E_{m,t} < 0 \}; \)
5: \( \text{Set } S = \{ \text{Group of MGs selling energy } | E_{m,t} > 0 \}; \)
6: The MGCC broadcasts Lagrange variables \( \lambda_i^i, \nu_i^i, \mu_i^i \) buyer set \( B \) and seller set \( S \);
7: \( q \leftarrow q + 1; \)
8: Each insufficient MG agent in \( B \) obtains optimal \( b_q \) by solving 30;\n9: Each surplus MG agent in \( S \) obtains optimal \( b_q \) by solving 31;\n10: Agents submit \( b_q \) and \( s_q \) th to MGCC; \n11: MGCC solves the allocation rule \( q_b^i \) and \( q_s^i \) by 24 and 25; \n12: MGCC updates the dual variables:\n13: \( \lambda_{i+1} = (\lambda_i - \sigma \frac{\partial L(\lambda)}{\partial \lambda_i}); \)
14: \( \mu_{i+1} = (\mu_i - \sigma \frac{\partial L(\mu)}{\partial \mu_i}); \)
15: \( \nu_{i+1} = (\nu_i - \sigma \frac{\partial L(\nu)}{\partial \nu_i}); \)
16: until The termination criterion is satisfied:\n17: \( q_{i+1}^b - q_{i-1}^s < \xi, q_{i+1}^s - q_{i-1}^b < \xi \)
18: return The payment rule \( P(x_i^b) \) and income rule \( I_j(x_i^s) \).

D. Analysis of Properties

In the following, we first show that the DDA scheme stated in Algorithm 1 converges to the optimal social welfare. Then, we prove that the DDA scheme satisfies the properties of individual rationality, (weak) budget balance, and strategy-proofness.

Theorem 1. The proposed double auction scheme converges and achieves optimal social welfare.

Proof. Recall that from lines 13-15 in Algorithm 1, we can observe that \( \sigma \) is the iteration step size, which is generally a very small number. Thus, we consider approximating the algorithm with a continuous time counter, and Lagrange variables are updated as:

\[
\frac{d\lambda_i}{dq} = \left( \sum_{j=1}^{S} q_i^{b}_{i,j} - b_i \right) + \frac{d\mu_j}{dq} = \left( \sum_{i=1}^{B} q_j^{s}_{j,i} - s_j \right) + \frac{d\nu_{i,j}}{dq} = (q_i^{b}_{i,j} - q_j^{s}_{j,i})
\]

(34)

To prove the convergence property of the distributed algorithm, we denote the Lyapunov function \( F(\lambda, \mu, \nu) \). If and only if \( F \) satisfies \( \frac{dF}{dt} \leq 0 \), we conclude that the DDA scheme converges to optimal social welfare with any initial condition. According to the scheme in [30, 31], the Lyapunov function [32] is denoted as:

\[
F(\lambda, \mu, \nu) = \sum_{i=1}^{B} \frac{(\lambda_i - \lambda_i^*)^2}{2} + \sum_{j=1}^{S} \frac{(\mu_j - \mu_j^*)^2}{2} + \sum_{i=1}^{B} \sum_{j=1}^{S} (\nu_{i,j} - \nu_{i,j}^*)^2
\]

(35)

By applying the update rule in Equation (34), we have:

\[
\frac{dF(\lambda, \mu, \nu)}{dq} = \sum_{i=1}^{B} (\lambda_i - \lambda_i^*) \frac{d\lambda_i}{dq} + \sum_{j=1}^{S} (\mu_j - \mu_j^*) \frac{d\mu_j}{dq} + \sum_{i=1}^{B} \sum_{j=1}^{S} (\nu_{i,j} - \nu_{i,j}^*) \frac{d\nu_{i,j}}{dq}
\]

(36)

(37)

Here, Equation (37) is derived based on the updated rules stated in Equation (34) and the character of projection \((\cdot)^+\).
Equation (38) holds due to the KKT conditions defined in Equations (19) and (20) of the SWM problem. After that, conducting several polynomial transformations, we can obtain Equation (39). Moreover, recall that the utility functions $U$ and $C$ are strictly concave and convex functions, respectively. Thus the derived functions of $U$ and $C$ are, respectively, monotonic decreasing and monotonic increasing.

As a result, Equation (39) is non-positive, the following equation $\frac{d F(x_1, y_1, z_1)}{d y_1} < 0$ holds, and the algorithm is able to converge and achieve optimal social welfare.

Theorem 2. The proposed double auction scheme achieves individual rationality.

Proof. Recall that individual rationality indicates that all participants obtain non-negative utility. Proving this property is equivalent to proof that, for each insufficient MG agent, we have

$$U_i(b_i^{s_i}) - \sum_{j=1}^{N} b_{ij}^{s_i} \geq 0.$$  \hspace{1cm} (41)

By utilizing the KKT condition of OAP and the IMO’s optimal condition (defined in Equation (24), Equation (41) can be expressed as:

$$U_i(q_i^{b_i}) \geq \sum_{j=1}^{N} q_{ij}^{b_i} \frac{\partial U_i(q_i^{b_i})}{\partial q_i^{b_i}}.$$  \hspace{1cm} (42)

Recall that $U$ is strictly concave. Then, it is bounded above by its first-order Taylor approximation as

$$U(y) \leq U(x) + U'(x)(y - x).$$  \hspace{1cm} (43)

Thus, the inequality (42) holds and the proposed double auction scheme achieves individual rationality.

Theorem 3. The proposed double auction schemes satisfies (weak) budget balance.

Proof. The property of being (weakly) budget balanced means that the auctioneer (MGCC) can obtain non-negative profit, which can be expressed on the basis of payment and income rules:

$$\text{Profit: } P_i(b_i^r) - I_j(s_j^r).$$  \hspace{1cm} (44)

By utilizing the condition stated in Equation (29) and the KKT condition in Equations (17) to 20, the profit in Equation (44) equals:

$$\sum_{i=1}^{N} \sum_{j=1}^{S_N} q_{ij}^{b_i} \lambda_i^* + q_{ij}^{s_i} \mu_j^*,$$  \hspace{1cm} (45)

which is always non-negative. Therefore, the proposed double auction scheme achieves (weak) budget balance.

Theorem 4. The proposed double auction scheme satisfies the property of being strategy-proof under the assumptions that the MGs agents are price takers.

Proof. According to the payment and income rules in Section IV-C, the insufficient and surplus MGs agents are induced to submit their current optimal bids and asks for purchasing or supplying electricity via solving the IMO and SMO problems. In doing so, the maximum social welfare of a system with MGs will be achieved. Thus, because the optimal bids and asks from agents yield the optimal social welfare, we can conclude that the proposed double auction is strategy-proof.

V. PERFORMANCE EVALUATION

We have conducted a performance evaluation to demonstrate the effectiveness of our proposed strategies. In the following, we first present evaluation methodology and then describe the evaluation results.

A. Methodology

In our evaluation, we consider a system based on a modified IEEE 33-bus test system, which is composed of five residential-based MGs operating in grid-connected mode. The basic structure is given in Fig. 4, observing that the five MGs are geographically close to each other. For the sake of simplicity, we assume that the distance between two neighboring nodes in the test system is one unit, and the rest can be done in the same manner. Each MG consists of residential houses and buildings in different sizes. In addition, local power generators, renewable energy resources, and storage batteries are included in the MGs. The detailed parameters related to local generation units are given in Table II. The local generator consists of two kinds of micro turbines and related parameters are listed in Table III. The first two rows indicate the index, micro turbine types, and coefficient parameters $a$, $b$, and $c$ of micro turbine generation cost. The last three rows indicate the index, minimum and maximum generation output of micro turbines, in kW, as well as the startup (SU) and shut down (SD) costs of micro turbines.

Furthermore, the penalty cost, efficiency of battery charging/discharging, and the cost of battery charging/discharging in the objective function defined in Equation (1) are settled as 0.01, 0.95 and 0.9, respectively. The load demands of individual MGs are all based on residential house loads. Loads are derived from a real-world data set obtained from Stanford University, consisting of meter readings from thousands of houses over 200 days [33].

![IEEE-33 bus based test system](image-url)
Table II: Number of local units

<table>
<thead>
<tr>
<th>Unit type</th>
<th>Micro Turbine</th>
<th>PV</th>
<th>WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>MG1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MG2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>MG3</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MG4</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MG5</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table III: Parameters of micro turbines

<table>
<thead>
<tr>
<th>Gen#</th>
<th>Type</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>MT</td>
<td>6</td>
<td>0.012</td>
<td>4.8 x 10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>MT</td>
<td>7</td>
<td>0.009</td>
<td>8.5 x 10^{-4}</td>
</tr>
</tbody>
</table>

In our simulations, we used the Monte Carlo-based method [36] to generate 1000 scenarios for each uncertainty parameter, and the probability of each scenario is 1/1000. Each scenario contains hourly load, real-time price, and wind and PV generated capacity. Notice that, in practice, a large number of scenarios will lead to increased computation time and complexity, while a small number of scenarios generated by the Monte Carlo-based method will result in the decline of accuracy. To balance computation time and accuracy, we used the fast-forward scenario reduction-based mechanism [37] to shrink 1000 scenarios to 10. All experiments were conducted on a computer with 3.5 GHz Intel Core i7-3770 CPU and 8 GB RAM.

To evaluate the effectiveness of the proposed Sto2Auc framework, the utility function $U_i(q_i)$ of the insufficient MGs is settled as a positive, increasing, and concave function:

$$U_i(q_i^b) = \sum_{j=1}^{SN} \log(q_{i,j}^b - dis \cdot q_{i,j}^b + 1),$$

where $dis$ is the distance between MG $i$ and $j$. Meanwhile, the cost function $C_j(q_j^s)$ surplus MGs is denoted as a positive, increasing, and convex function:

$$C_j(q_j^s) = a_j \sum_{i=1}^{BS} (q_{j,i}^s)^2 + b_j \sum_{i=1}^{BS} q_{j,i}^s,$$

where $a_j$ and $b_j$ are constant factors for each surplus MG.

In our simulation, we consider the following four metrics [38], [27]: (i) Operation cost of the system with MGs, which is expressed by the objective function in Equation (1). (ii) Social welfare in the secondary market, as denoted in the SWM problem stated in Equation (11). (iii) Demand insufficiency indicating the difference between electricity purchasing request $q_{bi,j}^s$ and permissive supply $q_{bi,j}^s$. (iv) Profit of MGCC, which is stated in Equation (44) and reflects the difference between the payment from insufficient MGs to the MGCC, and the income from the MGCC to surplus MGs.

B. IEEE 33-Bus Test System Case

Operation cost: Fig. 6 indicates the operation cost of the system with MGs, in the case where the bidding strategy is not in place, the case where the DBET scheme is in place and the case where the proposed double auction mechanism is implemented. Clearly, we can observe that, after the DBET and double auction mechanism for energy trading among local MGs is in place, the operation cost declines dramatically. In particular, the total operation cost for the system with MGs decreased by $542 when the double auction scheme is in place.
Fig. 6: Operation cost without bidding, with DBET mechanism and with double auction mechanism

This is because, after introducing energy trading among local MGs in the secondary market, the MGs surplus energy can be sold to either the utility grid and to the insufficient MGs. In this way, the surplus energy in the system with MGs can be allocated in a more cost-efficient manner. This is because the sum of MGs buying cost and selling revenue from or to each other are equal to zero in the objective function 1.

In addition, the operation cost of each MG is highly related to the configuration of its local units and energy usage preference. Table IV illustrates the detailed energy transactions and cost of the five MGs in a day when the proposed distributed double auction-based scheme is in place, in which values in the middle of the table indicate the amount of energy trading among sellers and buyers MG. As we can see that there exist some zeros in the table, such as the value in the first row of second column, which means that as MG2 contains surplus energy, MG1 does not win the bids ever, thus the transactions among these two MGs are zero.

Fig. 8: (a) Social welfare in four typical times, (b) Social welfare at T1=8:00 under different step size

**Social welfare:** To evaluate the effectiveness of the proposed DDA scheme, Fig. 8 (a) illustrates the social welfare of the system composed of MGs. We conduct the DDA algorithm over a 24 hour day. Due to space limitations, we show the optimal social welfare and iteration process at four typical times, 8:00, 12:00, 18:00 and 24:00, as an example. The gradation curves and the straight lines in the figure are social welfare achieved by the distributed algorithm in each iteration, and the optimal solution of SWM. As we can see from the figure, the social welfare gradually converges to optimal.

Table IV: Allocation (kWh) and cost ($) of 5 MGs

<table>
<thead>
<tr>
<th>Buyer</th>
<th>MG1</th>
<th>MG2</th>
<th>MG3</th>
<th>MG4</th>
<th>MG5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>1318</td>
<td>1195</td>
<td>458</td>
<td>1585</td>
<td>1326</td>
</tr>
</tbody>
</table>

TABLE IV: Allocation (kWh) and cost ($) of 5 MGs

**Fig. 7:** (a) Operation cost vs. penalty factor, (b) Operation cost vs. degradation factor

Furthermore, Fig. 7 (a) and Fig. 7 (b) illustrate the variation of the total cost in the system versus the penalty factor $\delta_{i,t}$ and battery degradation parameter. Fig. 7 (a) shows that a higher penalty parameter of the deviation between day-ahead scheduling and real-time delivery will result in a higher total cost for the system. Recall that a higher penalty cost will lead to a smaller deviation between day-ahead bids and real-time delivery of electricity, which reduces agility for the MGCC and agents to carry out energy trading among local MGs. This leads to a higher operational cost. Fig. 7 (b) shows that the total cost increases as battery degradation increases. The reason for this is a higher degradation parameter that makes the use of the benefits of energy storage in local MGs will not balance out the cost raised by its charging/discharging degradation.
evolution of double auction algorithm in Fig. 8(b). The figure indicates that a larger step size results in faster convergence to the optimal social welfare. In particular, the algorithm converges after 85 iterations when the step size is 0.1. In contrast, when the step size is 0.01, convergence requires nearly 600 iterations.

**Demand insufficiency:** In Fig. 9(a), we show the demand insufficiency, again at the four typical times used previously, expressed as $q^b_{i,j} - q^s_{i,j}$. Obviously, we can see that the gap between electricity demand and supply gradually converges to zero, meaning that the optimal amount of electricity submitted by the insufficient MGs can be eventually fulfilled. In addition, Fig. 8 and Fig. 9(a) jointly indicate that the proposed DDA scheme is able to achieve the optimal solution of energy trading among local MGs.

**MGCC Profit:** Furthermore, we evaluate the difference between insufficient MGs payments and income obtained by surplus MGs in Fig. 9(b), which is also regarded as MGCC profit. The figure illustrates that, as iteration continues, the total payment to the MGCC is larger than the income obtained from the MGCC to surplus MGs. On the other hand, the results shown in Fig. 9(b) also verify the property of (weak) budget balance, meaning that the auctioneer can obtain non-negative profit when the market achieves equilibrium.

VI. RELATED WORK

Energy management in the smart grid system consisting of MGs as a typical Internet of Things application has attracted growing attention, and a number of research efforts have been conducted [39], [40], [1], [2], [3], [4], [7], [8]. For example, Chaouachi et al. in [40] formalized intelligent energy management of MGs through artificial intelligence techniques, jointly with linear programming-based multi-objective optimization. Several research efforts have sought to address interactions among MGs [41], [42], [10]. For example, Nikmehr and Ravadanegh in [42] developed techniques to minimize the operation cost of grid-connected MGs. In this scheme, the energy dispatching between MGs was considered; however, the principle of energy trading among local MGs was not shown. Lee et al. in [41] developed a distributed mechanism for energy trading among MGs and a game theory-based scheme to achieve equilibrium.

Although none of the studies mentioned above addressed uncertainty issues in MGs directly, this topic has been the subject of many studies. Specifically, to address uncertainties in the smart grid, stochastic programming models have been used to manage energy resources in MGs [44], [45], [9], [46], [47], [48]. For example, Yang et al. in [44] proposed a stochastic framework, which considers uncertainties of wind power generation and statistical PEV driving patterns. Su et al. in [47] proposed a predictive control method based on stochastic models for MG operations and considered the uncertainty of PEV charging. In addition, Fathi et al. in [46] studied the energy scheduling problem in grid-connected MGs and considered uncertainties in energy demands. In their study, online stochastic iterations were applied to capture the randomness of load demands.

In addition, bidding and auction schemes for network resource markets and energy markets have been thoroughly studied [16], [49], [50], [51]. For example, Adika and Wang in [49] investigated a probabilistic bidding strategy to minimize the operation cost of MGs, and developed a water filling mechanism to allocate electricity. An optimal planning of the interconnected network of multi-MGs was discussed in [51], in which a probabilistic minimal cut-set-based iterative methodology was proposed to address the optimal planning issue. Regarding the design of auction mechanisms in the smart grid and MGs, far fewer efforts have been conducted. Moreover, the reverse and procurement auction schemes in [52] were studied in the smart grid. Notice that, in these schemes, either the number of buyers or sellers is one.

Distinct from existing research efforts, in this study, we have addressed the operational challenges posed by the uncertainties renewable energy in a system consisting of MGs, and have proposed a distributed double auction (DDA) scheme as a solution. Our proposed scheme can not only tackle various uncertainties in both supply and demand, but also considers local MGs as energy suppliers, allowing for efficient energy trading among MGs to minimize operation costs and improve energy delivery.

VII. CONCLUSION

In this paper, we addressed the optimal bidding problems arising from uncertainties due to renewable energy resources in the smart grid system with MGs that is a typical Internet of Things application in energy domain. Particularly, we proposed the Sto2Auc framework, primarily consisting of two components: (i) a two-stage stochastic programming process to derive the optimal real-time energy capacity of MGs, and (ii) a distributed double auction scheme which enables optimal energy trading among local MGs. We first formalized the optimal bidding problem as a two-stage stochastic programming process, which aims to minimize the operation cost and obtain the optimal delivery of electricity, while uncertainties from both supply and demand sides need to be considered. To enable energy trading among local MGs, we then proposed the secondary electricity market. To derive the optimal energy trading in the secondary market, we proposed a distributed double auction (DDA) scheme. Through theoretical analysis, we demonstrated that the DDA scheme satisfied the properties of (weak) budget balance, individual rationality, and strategy-proofness. We also conducted experiments on a modified IEEE-33 bus based system, and the results show that, when our proposed DDA scheme is in place, the operation cost of the system can be reduced significantly. In addition, our proposed DDA scheme achieves good performance with respect to social welfare, demand insufficiency, and MGCC profit.

REFERENCES


