An Efficient Hybrid Localization Scheme for Heterogeneous Wireless Networks

Zimu Yuan¹, Wei Li¹, Adam C. Champion², Wei Zhao³
¹Institute of Computing Technology, Chinese Academy of Sciences, China
²Department of Computer Science and Engineering, Ohio State University
³Faculty of Science and Technology, University of Macau, Macau, China
{yuanzimu, liwei}@ict.ac.cn, champion@cse.ohio-state.edu, umrector_zhao@umac.mo

Abstract—The ability to track and locate physical entities is a fundamental requirement for Cyber-Physical Systems (CPSSs), especially in an ad-hoc wireless environment. In Heterogeneous Wireless Networks (HWNs), hybrid localization schemes are needed due to the coexistence of both accurate and coarse measurement mechanisms. However, current localization schemes cannot fully satisfy HWNs’ accuracy requirements. Therefore, we propose a universal measurement metric called Direct Proportion Distance (DPD) that can leverage most existing measurement mechanisms such as TOA/TDOA, RSS, AOA, Link Diagnosis (LD) and Signal Coverage Detection (SCD). We also prove that DPD is directly proportional to the physical distance between two wireless nodes. Based on this metric, we present three new localization algorithms and compare them with classical methods. The experiments verify that our method performs better than previous localization algorithms when both accurate and coarse measurements are fully utilized.

I. INTRODUCTION

In Cyber-Physical Systems (CPSSs), wired or wireless networks serve as an infrastructure to integrate physical entities and computer systems. Localization technology plays an important role in these networks to locate and track physical entities, especially in a wireless environment. However, in Heterogeneous Wireless Networks (HWNs), different nodes have different types of measurement mechanisms. Some sophisticated nodes (e.g. mobile phones, GPS receivers, laptops) may have expensive devices that support accurate measurements such as TOA, TDOA, AOA and RSS [6-9] [10-11]. Other nodes (e.g. wireless sensors) may only have coarse measurement mechanisms such as Link Diagnosis (LD) [15-17] and Signal Coverage Detection (SCD) [25] [26]. This heterogeneity brings about two difficulties to accurately locating physical entities. First, due to coexistence of different measurement mechanisms, it is hard to universally express distances or angles between nodes. Second, due to the ad-hoc placement of network nodes, it is also hard to satisfy the rigid requirements of certain localization algorithms.

Currently, two approaches are used to solve the above problems. The first approach is to use hybrid measurements instead of a single measurement to enhance localization accuracy. Typical methods using this approach include TOA/AOA [27], TOA/RSS [28], etc. However, these methods only support limited types of measurements. Another approach is using a universal metric to support all measurement mechanisms. Hop-Based Localization (HBL) [22-24] is a typical method using this approach. The idea of HBL is to use hops to express distances among nodes. However, this method only supports coarse measurements such as LD and SCD. It cannot fully utilize accurate measurement mechanisms that already exist in HWNs to improve localization accuracy.

In this paper, unlike previous work, we propose a novel concept called Direct Proportion Distance (DPD). DPD can be used as a universal metric to take into account both accurate and coarse measurement mechanisms. The essential difference between DPD and hops is that the distance between two nodes expressed by DPD is directly proportional to their true distance. However, hops only indicate the connection relationship between two nodes. In this sense, hops can be seen as a special case of DPD when physical distances among nodes have equal lengths.

Applying the DPD metric to algorithms such as DV [18], RPA [19], and MDS-MAP [20], we present several DPD-based localization algorithms. Several practical issues related to localization accuracy are discussed. To verify the effectiveness and efficiency of these algorithms, we use simulations to compare our method’s performance with that of typical HBL algorithms. The experiments show that our approach performs much better than these algorithms.

The rest of the paper is organized as follows. Section II introduces related work. Section III provides the motivation and description of DPD. Section IV discusses several practical issues of DPD-based localization algorithms. Section V conducts experiments. Section VI concludes the paper.

II. RELATED WORK

In this section, we present a classification for existing localization schemes and discuss their feasibilities in HWNs. In fact, localization techniques have two independent steps. The first step is to measure physical quantities such as distances, angles, or linkages among wireless nodes. For a homogeneous network, network nodes use a universal metric to present physical quantities. For HWN, multiple measurement mechanisms coexist and hybrid measurements can be used to enhance localization accuracy. The second step is to use various localization algorithms (e.g. triangulation, trilateration, DV, etc.) to calculate locations. In this step, a single algorithm may be used to calculate locations. However, hybrid algorithms that integrate two or more algorithms can also be used to improve localization accuracy.

¹ For example, in the Trilateration algorithm, at least four nodes are required to work together to locate a single node. In addition, in order to obtain feasible accuracy of localization, more nodes are needed to eliminate measurement errors.
Based on the above discussions, we present a classification for existing localization schemes in Table 2.1. We can see that there are four different classes. Next, we briefly introduce each of these classes.

Table 2.1. A Classification of Localization Schemes

<table>
<thead>
<tr>
<th>Computation Model</th>
<th>Measurement Mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>[1, 4-10, 18, 21-26, 34, 35] DPD-M, DPD-MDS</td>
</tr>
</tbody>
</table>

For Class I, this type of methods uses a single measurement mechanism and a single localization algorithm to calculate locations. Most of existing localization schemes belongs to this type. The method in [4] first uses TOA measurement and then uses the trilateration technique to calculate locations. The method in [18] uses hops as a universal measurement metric and the DV algorithm to calculate locations.

For Class III, this type of methods uses a single measurement mechanism with hybrid localization algorithms. A typical scheme of this type is RPA [19]. This scheme first uses the TOA mechanism to measure distances. Then it uses the DV technique for the first round of calculation. Finally it uses the trilateration technique for the second round of calculation. This scheme combines two different localization algorithms to form a hybrid process. Their experiments show that this approach can improve localization accuracy.

For Class IV, this type of methods uses both hybrid measurements and hybrid localization algorithms. Some methods of this type may support two or more measurement mechanisms [27-30]. For example, in [27], wireless nodes can support both TOA and AOA measurement. Then a hybrid TOA/AAO algorithm is used to calculate locations. The work shows that such an approach can improve location accuracy over that of any single algorithm.

Our work belongs to both Classes II and IV. Based on a universal measurement metric called the Direct Proportion Distance (DPD), our scheme can support most existing measurement mechanisms. In addition, based on this metric, our scheme can also apply single localization algorithms (Class II) and hybrid localization algorithms (Class IV). In Table 2.1, we present three localization schemes: DPD-M, DPD-IM, and DPD-MDS, which use multilateration, iterative multilateration, and multi-dimensional scaling algorithms.

It is obvious that Class I cannot be applied to HWNs. For Class III, current methods of this type can only support coarse measurements by LD and SCD, which cannot achieve satisfactory localization accuracy. For Class IV, compared with our schemes, previous work only supports limited types of measurement mechanisms. Such schemes are not feasible to HWNs containing many nodes that only support coarse measurement mechanisms.

Different from previous localization schemes, our method can leverage both accurate and coarse measurement mechanisms. Based on the proposed universal measurement metric, our scheme can support both single and hybrid localization algorithms. To the best of our knowledge, the DPD-based localization scheme in this paper is proposed for the first time.

III. DPD-BASED DISTANCE MEASUREMENT

A. Challenges and Motivations

As mentioned, localization is a fundamental requirement of Cyber-Physical Systems in order to locate and track physical entities. Much work has been done for designing fast and accurate localization algorithms. However, accurately locating an entity accurately in a Heterogeneous Wireless Network (HWN) is a challenging problem. In fact, two reasons bring about great difficulties to accurately locate physical entities in HWNs. First, in HWNs, accurate measurement mechanisms such as TOA, TDOA, RSS, and AOA may co-exist with coarse measurement mechanisms such as Link Diagnosis (LD) and Signal Coverage Detection (SCD). Second, different localization algorithms such as triangulation, trilateration, DV, MDS, and others may also coexist depending on what measurement mechanisms they use. From Section II, we know that existing localization schemes cannot efficiently calculate locations in HWNs. Therefore, our motivation is to find an efficient way to enhance localization accuracy.

In this paper, we propose a new measurement metric called Direct Proportion Distance (DPD) that can leverage both accurate and coarse measurement mechanisms. DPD can be seen as a proportional indicator of the physical distance. That is, the longer the physical distance, the larger the DPD, and vice versa. In fact, DPD can unify various measurements into a uniform representation and take it as the input of localization algorithms. Since deriving the DPD is not straightforward, we introduce the idea of Relative Position (RP), which is the basis of DPD’s construction.

B. Relative Position (RP) of a Node Pair

1) Relative Position (RP)

![Figure 3.1. Relative Position of a Node Pair](image)

Here we give the definition of Relative Position (RP). We have a reference node \(a(x_a, y_a)\) and its two neighbors \(b(x_b, y_b)\) and \(c(x_c, y_c)\). Denote the physical distance between \((a, b)\) and \((a, c)\) as \(d_{ab}\) and \(d_{ac}\), respectively. Then we have

\[
d_{ab} = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \tag{3-1}
\]

In fact, HBL can be seen as a special case of the DPD metric by which the distance between any pair of neighbor nodes is simply treated as ‘1’.

4 Such algorithms are called DPD-based localization algorithms, which will be discussed in Section IV.
\[
\text{dist}_{ac} = \sqrt{(x_c - x_a)^2 + (y_c - y_a)^2} \quad (3-2)
\]

If \( \text{dist}_{ab} > \text{dist}_{ac} \), we call \( c \) closer than \( b \) to the reference node \( a \). Otherwise, if \( \text{dist}_{ab} \leq \text{dist}_{ac} \), then we say \( c \) is equal to or farther than \( b \) to \( a \).

Based on the above definitions, we introduce how to calculate RP via different measurement mechanisms.

2) Calculating RP via Different Measurement Mechanisms

Here we assume that any node can send/receive messages to/from its neighbors, i.e., all nodes support the SCD mechanism. Fig. 3.1 illustrates an example we will use here.

a) Calculating RP via TOA/ TDOA Measurement

For TOA measurement, we know that \( a \) can measure the round-trip distance between \((a, b)\) and between \((a, c)\). We have

\[
\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} = \frac{1}{2} c \cdot t_{ab}, \quad (3-5)
\]

\[
\sqrt{(x_c - x_a)^2 + (y_c - y_a)^2} = \frac{1}{2} c \cdot t_{ac}, \quad (3-6)
\]

where \( c \) is the speed of light and \( t_{ab} \) and \( t_{ac} \) are the measured signal round-trip travel time from \((a, b)\) and \((a, c)\). By this means, the equation in (3-3) and (3-4) can be determined. The calculation for TDOA measurement is similar.

b) Calculating RP via RSS Measurement

For RSS measurement, the received signal strength is measured and distance estimation is calculated by

\[
\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} = d_0 \left( \frac{P_{ab}}{P_{d0}} \right)^{-1/\eta} \cdot e^F \quad (3-7)
\]

\[
\sqrt{(x_c - x_a)^2 + (y_c - y_a)^2} = d_0 \left( \frac{P_{ac}}{P_{d0}} \right)^{-1/\eta} \cdot e^F, \quad (3-8)
\]

where \( F = (a^2/2)(a^2/\eta^2) \), \( a = \ln 10/10 \), \( P_{ab} \) and \( P_{ac} \) denote the received signal strength, \( P_{d0} \) denotes the signal strength at a reference distance \( d_0 \), \( \eta \) denotes the path-loss exponent, and \( a^2 \) denotes the random noise variance. Then, (3-3) and (3-4) can be determined to obtain \( a \)'s RP.

c) Calculating RP via Link Diagnosis

For Link Diagnosis, the link signature that captures the multipath characteristic and link quality between nodes is examined. With a set of classification methods such as Discriminant Analysis (DA), a can obtain the signature between itself and \( b \) and \( c \) separately. If \( a \) and \( c \) have more similar signatures, then, \( c \) is closer than \( b \).

d) Calculating RP via AOA Measurement

For AOA measurement, the direction of incoming signals is obtained. In Fig. 3.2, if the angle \( \angle bac \geq 90^\circ \), by the knowledge of geometry, we have following relations:

\[
\sqrt{(x_b - x_c)^2 + (y_b - y_c)^2} > \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \quad (3-9)
\]

\[
\sqrt{(x_b - x_c)^2 + (y_b - y_c)^2} > \sqrt{(x_c - x_a)^2 + (y_c - y_a)^2} \quad (3-10)
\]

Then both \( \text{dist}_{cb} > \text{dist}_{ab} \) and \( \text{dist}_{cb} > \text{dist}_{ac} \) can be established. However, when the angle \( \angle bac < 90^\circ \), we cannot decide the RP of \( b \) and \( c \). In this situation, RP will be calculated by the Signal Coverage Detection (SCD) mechanism.

\[\text{e) Calculating RP via Signal Coverage Detection}\]

Signal Coverage Detection (SCD) is the simplest measurement mechanism for wireless nodes in HWNs. If \( b \) is the neighbor of \( a \) but \( c \) is not, it is reasonable to obtain

\[
\text{dist}_{ab} < \text{dist}_{ac} \quad (3-11)
\]

If \( b \) and \( c \) are both neighbors of \( a \), we can assign

\[
\text{dist}_{ab} = \text{dist}_{ac} \quad (3-12)
\]

C. Direct Proportion Distance (DPD)

In this subsection, we will derive the concept of DPD. Based on the idea of RP, we first define a Reverse/Identical Pair. Then we present two theorems that are preconditions of the derivation of DPD. Finally, the physical meaning of DPD is explained.

1) Reverse/Identical Pair

Based on the definition of RP, we define two types of node pairs: Reverse Pair and Identical Pair. Suppose there are two neighboring nodes \( a \) and \( b \) with another two nodes \( c \) and \( d \) nearby. Without loss of generality, we assume \( \text{dist}_{ac} < \text{dist}_{ad} \) (the case \( \text{dist}_{ac} = \text{dist}_{ad} \) will be discussed in Theorem 3.1).

Here we have the following definitions:

**Definition 3.1. Reverse Pair.** In Fig. 3.2(a), we have \( \text{dist}_{ac} < \text{dist}_{ad} \) and \( \text{dist}_{cb} > \text{dist}_{bd} \). Here we call the nodes \( c \) and \( d \) a Reverse Pair for the node pair \( a \) and \( b \).

**Definition 3.2. Identical Pair.** In Fig. 3.2(b), we have \( \text{dist}_{ac} < \text{dist}_{bd} \) and \( \text{dist}_{bc} < \text{dist}_{bd} \). Then we call the nodes \( c \) and \( d \) an Identical Pair for the node pair \( a \) and \( b \).

2) Proportion Intersection Point

Based on Definitions 3.1 and 3.2, we give a theorem related to the concept of a Proportion Intersection Point. Denote the line segment between \( a \) and \( b \) as \( L_{ab} \) and the line segment between \( c \) and \( d \) as \( L_{cd} \). The perpendicular bisector of \( L_{cd} \) is denoted as \( B_{cd} \).

**Theorem 3.1.** In a 2D graph, \( B_{cd} \) must intersect with \( L_{ab} \) if \( c \) and \( d \) are a reverse pair in Fig. 3.2(a). The intersection point is called a Proportion Intersection Point (the point \( e \) in Fig. 3.2(a)). If \( c \) and \( d \) are an identical pair, \( B_{cd} \) and \( L_{ab} \) will not intersect (in the 3D case lines are treated as surfaces).

**Proof:** (for brevity) Assign two coordinates \((0, 0)\) and \((x, 0)\) to \( a \) and \( b \), respectively. Then, we only need to prove that the point \( e \) with coordinates \((x_e, 0)\) must have \( 0 < x_e < x \) if \( c \) and \( d \) are a reverse pair. Otherwise \( x_e < 0 \) or \( x_e > x \) if \( c \) and \( d \) are an identical pair.

Nodes \( c \) and \( d \) have coordinates \((x_c, y_c)\) and \((x_d, y_d)\) respectively. The coordinates of \( e \) can be written as \( x_e = x_c - y_c(x_d - x_c)/(y_d - y_c) \) or \( x_e = x_d - y_d(x_d - x_c)/(y_d - y_c) \). If we have a reverse pair, the conditions \( \text{dist}_{ac} < \text{dist}_{bd} \) and \( \text{dist}_{bc} > \text{dist}_{bd} \) exist and we can easily prove \( 0 < x_e < x \) by algebraic substitutions. If we have an identical pair, the

\[\text{Note that the accuracy of DPD-based localization algorithm may suffer from this simplification.}\]
conditions $\text{dist}_{ac} < \text{dist}_{ad}$ and $\text{dist}_{ae} < \text{dist}_{bd}$ exist and we can prove $x_a < 0$ or $x_a > x$ similarly. When we have $\text{dist}_{ac} = \text{dist}_{ad}$ (or $\text{dist}_{bd} = \text{dist}_{eb}$), we can directly derive from the perpendicular bisector theorem that $B_{cd}$ must cross the node $a (0, 0)$ or the node $b (x, 0)$.

3) Derivation of DPD

Based on Theorem 3.1, we provide Theorem 3.2, which is the foundation of DPD. In fact, Theorem 3.2 will explain why DPD can be a proper indicator of the physical distance among nodes. Before we give the proof of Theorem 3.2, we provide some basic definitions first.

Supposing there are a total of $N$ nodes (including nodes $a$ and $b$) randomly distributed in the area around $a$ and $b$. Let $S$ be the size of this area. Denote the total number of pairs $n_t$ and choosing 2 nodes from $N$, we have $n_t = N(N - 1)/2$. Let $n_r$ be the total number of reverse pairs in $n_t$ node pairs.

Since $n_t$ pairs have $n_t$ perpendicular bisector lines, the area can be divided into $n_p = (n_t^2 + n_t + 2)/2$ small pieces [33]. Thus, the expected diameter is proportional to the square root of the small pieces' expected size $E[s_p]$. As a result, $E[d_p] = a E[s_p] = a \sqrt{S/n_p}$, where $a$ is a constant area shape factor as nodes are assumed to be randomly distributed in this neighboring area. Based on the above definitions, we have the following theorem:

**Theorem 3.2.** Given the node pair in Fig. 3.2, $\text{dist}_{ab}$ is the physical distance between $a$ and $b$. Then, $\text{dist}_{ab}$ is directly proportional to $n_r$.

**Proof:** As proved in Theorem 3.1, the perpendicular bisector line of a reverse pair must pass the line segment $L_{ab}$. The distance $\text{dist}_{ab}$ can be calculated as $n_r \cdot E[d_p]$ (the total number of reverse pairs distinguished from total $n_t$ pairs times the expected diameter of small pieces). Then we get an approximation of the distance of $L_{ab}$:

$$\text{dist}_{ab} = n_r \cdot E[d_p] = a n_r \sqrt{S/n_p} = \sqrt{\frac{a}{2 \pi \rho S}} \times \frac{a}{\rho} \left( \frac{25}{n_t^2 + n_t + 2} \right) \quad (3-13)$$

In a 2D case, we have $N = \rho S$, where $\rho$ denotes the density of nodes in this area. Using $N$ to substitute $S$ and $n_t$, we can rewrite (3-13) as

$$\text{dist}_{ab} = \varphi n_r \sqrt{\frac{a}{2 \pi \rho S}} \times \frac{a}{\rho} \left( \frac{25}{n_t^2 + n_t + 2} \right) \quad (3-14)$$

where $\varphi = 2 \sqrt{2} \pi \rho S$.

In a 3D case, we have $N = \rho V = \frac{4}{3} \sqrt{\pi \rho S^2}$ and we can rewrite (3-13) as

$$\text{dist}_{ab} = \varphi n_r \sqrt{\frac{a}{2 \pi \rho S}} \times \frac{a}{\rho} \left( \frac{25}{n_t^2 + n_t + 2} \right) \quad (3-15)$$

where $\varphi = \frac{3}{4 \sqrt{\rho}} \sqrt{\frac{a}{\rho}} \left( \frac{25}{n_t^2 + n_t + 2} \right)$.

We use an example showed in Fig. 3.3 to explain Theorem 3.2. The local area is divided into small pieces by the perpendicular bisectors of both the identical and reverse pairs. There are a total number of $n_r = 5$ perpendicular bisectors of reverse pairs and the intersection points are $n_1, n_2, n_3, n_4,$ and $n_5$ respectively. That is, the line segment $L_{ab}$ passes $n_r + 1 = 6$ small pieces, which are $P_{an_1}, P_{n_1n_2}, \ldots, P_{n_5b}$. Actually, we have $\text{dist}_{ab} = \text{dist}_{an_1} + \text{dist}_{n_1n_2} + \ldots + \text{dist}_{n_5b}$. Since we cannot get the exact values of $\text{dist}_{an_1}, \text{dist}_{n_1n_2}, \ldots, \text{dist}_{n_5b}$, here we calculate expected diameters of small pieces $E[d_p]$ instead.$^7$ We can estimate $\text{dist}_{ab}$ by $\text{dist}_{ab} = n_r \cdot E[d_p]$.

**Figure 3.3. Illustration of Theorem 3.2**

**Definition 3.3. Direct Proportion Distance (DPD).** Based on Theorem 3.2, we provide the following definition as a metric to indicate the physical distance between $a$ and $b$:

$$\text{DPD}_{ab} = \frac{\text{dist}_{ab}}{\varphi} \quad (3-16)$$

It is easy to see that $\text{DPD}_{ab}$ is proportional to the physical distance $\text{dist}_{ab}$ (i.e. $\text{DPD}_{ab} \propto \text{dist}_{ab}$). Thus, DPD can be used to indicate the actual physical distance proximity between $a$ and $b$.

**Remarks.** In the above definition, it is necessary to obtain $N$ (total neighboring nodes of $a$ and $b$) and $n_r$ (the total number of reverse pairs in neighboring area of $a$ and $b$) to calculate DPD$_{ab}$ by (3-14), (3-15), and (3-16). Then, we need to judge if a pair of neighboring nodes of $a$ and $b$ is a reverse pair. To make this judgment, we need to compare RP of $a$ and $b$. For example, we need to decide which of $c$ and $d$ is nearer to $a$ (or to $b$). Note that in the above calculation, we need not measure the exact distance such as $\text{dist}_{ac}$.

IV. PRACTICAL ISSUES IN DPD-BASED LOCALIZATION ALGORITHMS

A. DPD-Based Localization Algorithms

Fig. 4.1 shows the relationship between our proposed DPD metric and existing localization algorithms. We can see that the DPD metric acts as a middle layer between measurement mechanisms and localization algorithms. That is, the DPD metric unifies various measurements into a uniform representation that can be taken as the input of existing localization schemes such as DV [18], RPA [19], MDS-MAP [20], and others [34] [35]. DV, RPA and MDS-MAP are typical localization schemes that use computation models such as multilateration, iterative multilateration, and multi-dimensional scaling (MDS), respectively, to calculate all nodes’ locations.$^8$

$^7$ Since two ends $\text{dist}_{an_1}, \text{dist}_{n_5b}$, are residuals, we consider the value of $\text{dist}_{an_1} + \text{dist}_{n_5b}$ is approximately equal to the expected diameter $E[d_p]$.

$^8$ At now, DPD input for algorithms are still not quite accurate, so the triangulation model cannot be adopted.
these typical algorithms to form DPD-based algorithms. Since the DV, RPD, and MDS-MAP algorithms can use physical distances or hop-based distances as inputs, we can directly apply the DPD metric to these algorithms without any modifications. To distinguish our approach from others, we denote our DPD-based algorithms DPD-M, DPD-IM, and DPD-MDS. Similarly, the hop-based algorithms are denoted DV-Hop, RPA-Hop, and MDS-Hop respectively. In Section V, we will compare the performance of DPD-based algorithms with Hop-based algorithms.

![Figure 4.1. DPD-based Localization Architecture](image)

After presenting DPD-based localization algorithms, we need to answer another question: what is the quality of DPD-based algorithms in terms of localization accuracy? From knowledge of measurement error analysis, we know that the accuracy of localization algorithms depends on the measurement error and error propagation of localization algorithms. Next, we will investigate how to provide more accurate DPDs to improve localization accuracy.

From Section III, we know that DPD is calculated by

\[ N_{ab} = |S_a \cup S_b| \] (i.e., the total number of neighbors of \( a \) and \( b \)) and \( n_r \) (the number of reverse pairs). Next, we will study how these two parameters affect the accuracy of DPD and select optimal configurations for them.

### B. Effect of Reverse/Identical Selections on DPD

![Figure 4.2. Selecting Reverse/Identical Pairs with Different Strategies](image)

In this part, we study how to choose node pairs to make DPD more linearly proportional to the physical distance. Fig. 4.2 compares DPD values (the vertical axis) with true physical distances (the horizontal axis). In Fig. 4.2(a), we select all node pairs (both neighboring pairs and non-neighboring pairs) to calculate DPD\(^9\). In Fig. 4.2(b), we select all neighboring pairs to calculate DPD\(^10\).

From Fig. 4.2(a), we can see that DPD calculated by neighboring pairs (in red dots) is approximately directly proportional to the physical distance compared with DPD calculated by non-neighboring pairs (in blue dots). Dashed lines in Fig. 4.2(b) show the spread pattern of DPD calculated by neighboring pairs. From both Fig. 4.2(a) and 4.2(b), we can see that the choice of neighboring pairs to calculate DPD is more appropriate for DPD-based localization algorithms.

### C. Effects of Measurement Errors on RP

Due to the limitation of measurement mechanisms, we may face the uncertainty problem when calculating RP. That is, for a node pair \((b, c)\) with a reference node \(a\), \(d_{stab} > d_{stac}\). However, in practice, we may get \(d_{stab} < d_{stac}\) due to imprecise measurements. To study how measurement errors affect the correctness of RP, we introduce a concept called the estimated reverse pair count and denote it as \(n'_r\). The definition of \(n'_r\) is formalized as

\[ n'_r = \sum C_{c,d}, \] (4-1)

where \(c, d \in S_a \cup S_b\) and \(C_{node, node} \in [0, 1]\). Here, \(C_{c,d}\) is the confidence to determine reverse pairs. Followings are the meanings for different \(C_{c,d}\):

1) \(C_{c,d} = 0\) indicates a convinced identical pair;
2) \(C_{c,d} = 1\) indicates a convinced reverse pair;
3) \(C_{c,d} \in (0, 1)\) indicates the uncertainty of an identical pair or reverse pair.

Based on the above definitions, we introduce the ratio \(Ratio_{false,RP}\) to measure the uncertainty of RP:

\[ Ratio_{false,RP} = \frac{\ln n'_r - n_r}{n_r} \times 100\% \] (4-2)

In (4-2), if \(n'_r\) is far less than \(n_r\), \(Ratio_{false,RP}\) is large. Then we say the uncertainty of RP is high (and vice versa). Assigning different values of \(C_{c,d}\) (between 0 and 1), we can provide different levels of uncertainty of HWNs. In this paper, we investigate three levels of uncertainty to distinguish HWNs with different measurement accuracies:

1) **Low Uncertainty.** We consider accurate measurement mechanisms such as TOA/TDOA, AOA and RSS. If the network is dominated by these measurement mechanisms, the uncertainty is low.
2) **Middle Uncertainty.** This level means that accurate and coarse measurement mechanisms are nearly equally available in HWNs.
3) **High Uncertainty.** This level indicates that most wireless nodes are only equipped with coarse measurement mechanisms such as LD or SCD.

Based on the above definitions, in Section V, we will investigate the efficiency of DPD-based localization algorithms on different levels of uncertainty. Actually, experiments show that our methods perform much better than others when the HWN uncertainty of is low.

\[^9\] Here short physical distances in the horizontal axis indicate that neighboring pairs are selected.

\[^10\] The network setting for this is a 2D network of 200 nodes randomly deployed in 500 m × 500 m square area. Subject to unequal transmission power and environmental interference, the radio range of each node is randomly selected between 50 m and 200 m.
V. PERFORMANCE EVALUATION

In this section, we will compare DPD-based localization algorithms (DPD-M, DPD-IM and DPD-MDS) with HBL algorithms. Before we present experiments, we first introduce some performance metrics and configurations.

A. Performance Metrics and Experiment Configurations

In our experiments, we use the Average Positioning Error (APE) to measure localization accuracy. That is, we have

\[ APE = \frac{1}{N} \sum_{i=1}^{N} \sqrt{(x_i^e - x_i)^2 + (y_i^e - y_i)^2}, \]

where \((x_i^e, y_i^e)\) is the calculated location and \((x_i, y_i)\) is the true location. \(N\) is the total number of nodes.

For the node pair selection issue in Section IV-A, we select neighboring node pairs in our experiments to calculate RP. For the uncertainty issue in Section IV-B, we set \(R_{\text{all} \times \text{RP}}\) to 10%, 20% and 30% to study the performance of our methods under different levels of uncertainty.

A square area \(M(m) \times M(m)\) with \(N\) nodes is randomly deployed. \(Q\) beacon nodes are randomly selected from all nodes. We regenerate the network \(R\) times and take the average confidence results. By default, we have \(M(m) = 500\), \(N = 200\), \(Q = 8\), and \(R = 100\). The log-distance path loss model [10] is used. The transmission power of each node is randomly chosen from -20 dBm to 0 dBm. The sensitivity threshold is randomly chosen from -100 dBm to -80 dBm. The path-loss factor \(\eta\) is set to 4. The random fading noise \(X_\sigma\) is set to 6 by default.

B. Experiments and Analysis

Table 5.1 gives notations for DPD-based localization algorithms. The terms “Low”, “Mid”, and “High” indicate levels of low, middle, and high uncertainty respectively.
### TABLE 5.1 NOTATIONS OF ALGORITHMS IN FIG. 5.1 – FIG. 5.3

<table>
<thead>
<tr>
<th>Localization Algorithms</th>
<th>Level of Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>DPD-M (DV)</td>
<td></td>
</tr>
<tr>
<td>DPD-Low-M</td>
<td></td>
</tr>
<tr>
<td>DPD-Mid-M</td>
<td></td>
</tr>
<tr>
<td>DPD-High-M</td>
<td></td>
</tr>
<tr>
<td>DPD-IM (RPA)</td>
<td></td>
</tr>
<tr>
<td>DPD-Low-IM</td>
<td></td>
</tr>
<tr>
<td>DPD-Mid-IM</td>
<td></td>
</tr>
<tr>
<td>DPD-High-IM</td>
<td></td>
</tr>
<tr>
<td>DPD-MDS (MDS)</td>
<td></td>
</tr>
<tr>
<td>DPD-Low-MDS</td>
<td></td>
</tr>
<tr>
<td>DPD-Mid-MDS</td>
<td></td>
</tr>
<tr>
<td>DPD-High-MDS</td>
<td></td>
</tr>
</tbody>
</table>

1) **Experiments on Changing of Beacon Numbers**

The number of beacon nodes increases from 3 to 20 with step 1. From Fig. 5.1, we can see that increasing beacon nodes helps improve the accuracy of all methods. For all cases, our method performs better than HBL. In particular, our method performs 32.5%, 20.6%, and 6.8% better than the MDS-Hop algorithm for low, middle, and high uncertainty. Our method also shows significant improvements compared with the DV and RPA algorithms.

2) **Experiments on Changing of Node Density**

The number of nodes changes from 50 to 400 with step 50. From Fig. 5.2, we see that our methods have a similar trend to HBL methods. In addition, our methods are better than HBL methods in terms of localization accuracy. In comparison with the MDS algorithm, our method shows 29.6%, 17.3%, and 2.9% improvement for low, middle, and high uncertainty respectively. In addition, our method gets more significant improvements compared with the DV and RPA algorithms.

3) **Experiments on Changing of Area Size**

Both the area length and width are enlarged from 200 m to 1000 m in 100 m steps. The number of nodes proportionally with the area size to keep the node density constant. The beacon number remains at 8 when the area size changes. In Fig. 5.3, for all algorithms, the positioning accuracy gets worse as the scale of the network increases. Nevertheless, our method performs 35.7%, 20.4%, and 5.6% better than the MDS-Hop algorithm for low, middle, and high uncertainty, respectively. Our method performs significantly better than the DV-Hop and RPA-Hop algorithms.

### VI. CONCLUSIONS AND FUTURE WORK

In this paper, we propose a new metric called DPD that well suits localization in heterogeneous wireless networks in CPSs. The derivation of DPD is based on indirect measurement mechanisms that unify various measurement methods to a single relation called **Relative Position**. The directly proportional relationship between DPD and physical distances is proven by two theorems. The efficiency of DPD-based localization algorithms is verified via comparison with hop-based localization algorithms in various environments.

### Acknowledgment

This work is supported in part by the National Basic Research (973) Program of China (Grant No.2011CB302502, 2011CB302803), and the major national science and technology special projects (Grant No.2010ZX03004-003-03).

### REFERENCES


