

## INTERNATIONAL REAL ESTATE REVIEW

2005 Vol. 8 No. 1: pp. 44-82

# Tenant Management and Lease Valuation for Retail Properties: A Real Options Approach\*

### **Takeaki Kariya**

Graduate School of Business, Meiji University, KIER, Kyoto University and RIETI, Japan; kariya@kier.kyoto-u.ac.jp; <http://www.kier.kyoto-u.ac.jp/fe-tokyo>

### **Yasuyuki Kato**

KIER, Kyoto University, Nomura Securities Co. Ltd., Japan

### **Tomonori Uchiyama**

Nomura Securities Co. Ltd., Japan

### **Takashi Suwabe**

Goldman Sachs Co. Ltd., Japan

In this paper, we formulate a tenant management problem for retail properties, such as shopping centers, provide an analytical framework for deriving the probability distribution of the sum of discounted future cash flows stochastically generated through tenant management, and find an optimal lease agreement structure and strategy for tenant-replacement management. More specifically, we formulate the problem of valuing the net present value of future net income from a retail property with tenant management and provide a valuation model for management decision making. The income fluctuates with market rent variations and management processes. In our framework, a property manager is required to choose an optimal mix of fixed rent and variable rent linked to tenant sales, and one of two tenant-replacement rules for return and risk enhancement. Finally, we provide an optimal strategy for this problem using Monte Carlo simulation, through which we value the real options of adopting an optimal strategy for percentage rent and tenant replacement made available by the New House Lease Law in

---

\* The authors are grateful to the referees for their valuable comments. This is a joint research project with the Applied Financial Engineering Division of Nomura Securities Group at KIER. Kariya's portion is supported by Japan Science Promotion Foundation C(2)16530139.

Japan.

## **Keywords**

tenant management; DCF valuation; shopping center; real options; tenant replacement rule; percentage rent

## **Issues and Objectives**

The key aspects of commercial property management are property management (narrowly defined) and tenant management. Property management involves managing the physical structures, the technical functions of the buildings, and other tangible aspects of the property so as to protect the potential value of the property, attract tenants, and thereby increase cash flows. Tenant management, by contrast, involves managing the combinations of tenants, the structure of the lease agreements, and other intangible aspects that also affect the value of a property. In general, property management in the broad sense encompasses both aspects noted above.

In tenant management of a retail property, such as a shopping center, managers need to develop an effective business model for tenant location, tenant selection, and the structure of lease agreements so as to create value, increase the brand value of the property rentals, and achieve a long-term stability in the value of discounted cash flows given various uncertainties. Colwell and Munneke (1998) examined the value-enhancing aspects of percentage leases and explored the mechanisms of tenant mix, risk sharing and rent discrimination through which the value is created. In particular, Miceli and Sirmans (1995) considered the problems of leasing arrangements between a shopping center landlord and individual stores in the presence of inter-store externalities and showed that the key element of achieving the goal is the ability of the landlord to cancel the leases of stores whose sales fail to achieve a target level (see also Wheaton (2000)). On the other hand, Wheaton and Torto (1995) studied on the relationship between regional shopping center rental rates and retail sales and found that rental rates increased more rapidly than retail sales did between 1968 and 1993. Chun, Eppli and Shilling (1999), meanwhile, built on models developed by Benjamin, Boyle and Sirmans (1990, 1993) and Miceli and Sirmans (1995), but treated base rents and percentage rents as functions of sales, distinguished between fixed and percentage leases, and incorporated lagged effects. These studies analyzed the relationship between rents and sales for various retail formats from a macro supply-demand perspective.

In this paper, in a line with Miceli and Sirmans (1995), we consider the management problem for owners of shopping centers and other retail properties with options of percentage rents and tenant replacement rules, provides an analytical framework for modeling future cash flows under uncertainties in market rents and tenant's sales, and in terms of return (expected discounted cash flow) and risk (lower semi-deviation) find optimal lease agreement structures and tenant replacement rules through simulations. The percentage lease option model proposed in this paper has different structure from those commonly adopted in the US markets, where the percentage rent is a sum of a flat base rent plus a variable rent (overage), which is computed as a proportion of the sale revenue above the "breakpoint." In our case the percentage rent is defined as

$$(1 - \alpha)[\text{market rent}] + \alpha[\text{sales}] = [\text{market rent}] + \alpha[\text{sales} - \text{market rent}],$$

where the market rent is fixed for each lease term. Hence if the inside of the second term in the right side is negative, owners lose rents relative to the fixed case, implying a risk-sharing scheme. In developing the analytical framework we adopt the discrete time approach in Kariya and Liu (2003).

Another aspect of this paper is a policy assessment of the new House Lease Law in Japan that was in effect in 2000, which is abbreviated as NHLL in the sequel. New deregulatory provisions increase the value of the various real options. The new 2000 law is no exception, and under the NHLL, lessor can have a real option in the form of a contractual right to replace lessees. It gives various real options to retail property managers. Using the analytical model in this paper, we value the real option of adopting percentage rents linked to sales and tenant-replacement rules through simulations. Concerning the valuation of lease contracts, Grenandier (1995, 1996) proposed a unified real options model in a continuous-time game-theoretic equilibrium analysis and treated the case involving default. In our model, we assume for simplicity of the option structure in contracts that tenants are given no option to choose a lease structure and default, which avoids a game-theoretic nature of the contracts. This assumption applies to the case where a retail property of interest attracts tenants and a lease period is short. In Japan a typical lease period is 2 to 3 years. In addition, our model that has two stochastic factors of market rent and tenant's sales is discrete-time and path-dependent and our framework of decision making involves an explicit risk element as in the portfolio theory. This differentiates our model from the real option model using continuous-time diffusion processes possibly with a concept of no-arbitrage or optimizing an expected utility.

The model with two stochastic variables is robust in valuing retail leases. Optimal strategies for retail owners are derived in terms of return and risk

via simulations where tenant replacement cost is considered. The results correspond to an efficient frontier analysis in the Markowitz portfolio theory, but in our case a return-risk relation is optimized with respect to the proportion  $\alpha$  of percentage rent and a threshold in a tenant replacement rule based on a function of tenant's past sales. Even if a heavy tenant-replacement cost is imposed, the NHLL is shown to improve on return and risk over the case of the old law by 21% and 24 % respectively, implying that the New Law greatly creates value and reduces risk simultaneously.

### ***Three objectives***

The first objective of this paper is to provide an analytical framework for the tenant management problems that can be regarded as a combination of formulating rules for replacing tenants and structuring lease agreements that create value. As our valuation framework for quantitatively determining the present value of future rent cash flows, we propose a dynamic discounted cash flow (DDCF) model that takes into account differences in tenant-replacement rules and the structure of the lease agreements in terms of fixed versus variable rents. In the framework the value of a retail property is in fact a probability distribution as the distribution of possible DDCF values since DDCF-based values are subject to stochastic variations over a time horizon in the future. The DDCF distribution is derived using a Monte Carlo simulation, because of its nonlinear nature. Using the mean and the lower semi-deviation of the distribution we formulate an optimization problem in terms of return and risk. The management problem can thus be regarded as one of solving the optimization problem that leads to valuing retail property under a specific business model.

The second objective is to find optimal combinations of tenant-replacement rules and lease agreement structures through a comparative analysis of alternative lease agreement structures and tenant-replacement rules. The mean of the distribution is typically known as the DCF expected value of the distribution. In our analysis, taking lower semi-deviation as a risk measure, via simulations we optimize our risk-return problem with respect to percentage rents and tenant-replacement rules in association with the problem of valuing a real option described below.

The third objective is to conduct a policy assessment of the NHLL 2000 in Japan by valuing the real option that the law provides. The value of this option is the difference between the value of the option when it is used most effectively and the value when retail property without tenant-replacement rights is used. With a tenant-replacement option, a percentage (variable) rent option is created. Under the old Law for Land Lease and House Lease (abbreviated as OHLL), no provisions existed for tenant-replacement rights,

which made it risky and thereby difficult to use percentage-of-sales rental contracts. With a tenant-replacement option, however, property management companies are able to actively use such lease agreements and add value.

It is remarked that until the introduction of the NHHL the OHLL has given lesses the right to renew the rental contract at the end of each contract term and has not allowed lessors to reject the renewal even if an adjustment for rent can be made. The contracts made under the OHLL still carry this right. Note that in Japan a lease period is typically 2 to 3 years.

To pursue these objectives, in Section 2 of this paper, the retail property management issues of tenant selection and lease agreement structure are discussed. In essence, the issue for retail property managers can be thought of as developing common incentives with tenants to create value, given a certain business model. In Section 3, we develop one contract structure that provides such common incentives through a combination of fixed and percentage rents, and frame the tenant selection issue by focusing on sales as they relate to ability to attract and retain customers. We propose two rules in which tenants that are unable to meet certain sales conditions within the contract period are replaced. In this way, the value of the retail property can be maintained and brand value can be established. In addition we formulate an optimization problem with respect to percentage rents linked to sales and tenant replacement rules. In Section 4, we develop a specific analytical model based on uncertainties that cause real estate values to fluctuate, specifically variation in market rental rates and variation in tenants' sales. The market rental rate determines each fixed rental rate for each contract period, and is generally related to economic conditions as well as the development trends and competitive characteristics of the region in which the property is located. In this paper, however, we assume a log DD (discrete time diffusion) process for analytical simplicity. Likewise, we assume a log DD process of the same type for the change in tenants' sales. For the drift of the models, though, we assume an exponential smoothing model that gradually changes in response to changes in its own previous trend, to take into account the non-Markovian (path-dependent) characteristics of the actual changes. As strategies for dealing with uncertainties that property managers actually face, these two models form a basis for describing tenant-replacement rules and contract structures that combine fixed and percentage rents.

Many Monte Carlo simulation results are carried out in Section 5 to firstly analyze the characteristics, analytical capability and phenomenon-describing capability of the model for comparing two tenant-replacement rules for an optimal mix of fixed and variable rents for each rule, to secondly find optimal strategies for owners of retail properties, and to thirdly value the real

options of the NHLL in the case of no tenant-replacement cost. For this purpose, we establish four cases:

- Case A: The core case of fixed rents and no tenant-replacement provisions
- Case B: A mix of fixed and percentage rents, and no tenant-replacement provisions
- Case C: A mix of fixed and percentage rents, and full tenant-replacement provisions
- Case D: A mix of fixed and percentage rents, and (I) tenant-replacement rules based on the average change in sales, and (II) tenant-replacement rules based on the level of sales

The comparison is based on the risk and return of the probability distribution of the DDCF values.

Case A corresponds to the base case of the OHLL. In Cases B, C, and D assuming no cost for tenant replacement, optimal solutions for the tenant management and lease structure are solved relative to the case of the OHLL and the value of the real options made available by the NHLL is evaluated with the optimal solutions. Cases B and C show as two extreme cases effectiveness of a mix of fixed and percentage rents with no tenant-replacement and full tenant-replacement respectively. These cases also show the robustness of the model. The results for these four cases are based on a volatility for the sales process of 20%. In addition, we test

Case E: Case D, but with a volatility for the sales process of 10%.

In Section 6, taking into account the cost of tenant replacement, optimal solutions for the problems stated above are numerically derived for different costs when percentage rate for variable rents is 50%. In Section 7 the efficient frontier of risk (lower semi-deviation of DDCF property distribution) and return (expected value of DDCF property distribution) with various tenant-replacement cost is derived for a general case, and an optimal percentage rent and tenant-replacement is obtained with the cost of 24-month rents, which also gives a valuation of the real option.

## **Perspectives on Managing Retail Property**

The category of retail property we consider in this section is shopping centers. The tenant management issues can be specified as follows:

- 1) The business concept of the shopping center, and a business portfolio of tenants and their locative allocation in the shopping center
- 2) Tenant selection, given a certain business portfolio and tenant locations, and lease agreement structures

The first issue concerns the desired customer segment for the shopping center; the positioning of the shopping center, in terms of grade, function, and regional role, as part of the core business model; and the determination of a business portfolio of tenants for the business model and their locations. This issue is a difficult one that has much to do with whether a shopping center management business is successful. A typical mall in the United States features high-end department stores, with spacious sales floors; Sears, J.C. Penney, and other general department stores that sell inexpensive household goods and cater to the middle class; specialty footwear and clothing stores; and McDonald's and other food establishments. This retail arrangement stems from a business concept, and may be attractive to consumers. Based on the location of the types of businesses and the selection of tenants attractive to the target customer segment, the objective is to bring in customers repeatedly, have them stay for long periods and spend money, and generate externalities for the other tenants in terms of profits. We do not consider in this paper how to come up with an optimal combination of different businesses of tenants and their locations.

Our objective is the second issue above, namely providing a framework for defining the problem and a cash flow valuation model. Specifically, we address issues concerning the structure of the lease agreements and the change of tenants based on sales, on the assumption that the management company has the right to ask tenants to leave. Hence, we

- 1) Use a combination of fixed and percentage rents for the lease agreement, and
- 2) Consider tenants' sales growth rates and variability of sales as tenant characteristics

The partial use of percentage rents is important in that they can provide common incentives for the property management company and the tenants, and encourage both of them to be interested in how well the tenants do. In addition, strong sales mean a strong ability to attract customers, a factor that leads to externalities also benefits the businesses of other tenants. Sales data are readily available and in fact are often stored in the computer of managers.

The structure of the lease agreement may differ depending on the tenant's type of business, as it relates to the business concept of the shopping center, and on the positioning of the tenants. Coffee shops, for instance, may not have particularly significant sales or much variability in sales, but are still an important type of retail business for shopping centers because of their ability to draw customer traffic. A portfolio consisting of such tenants and those in businesses with relatively significant sales and variability of sales can be put together to match the desired business concept. For structuring lease

agreements, an analytical framework for considering the choices of combinations of fixed and percentage rents for different types of businesses is needed. In this paper, we provide such an analytical framework and a specific model, quantitatively compare DDCF probability distributions via simulations involving different combinations of fixed and percentage rents and solve optimization problems with respect to percentage rents and tenant replacement rules.

The second issue involves the valuation of shopping center properties as a probability distribution of the DDCF value when tenants are replaced on the basis of sales, given lease agreements that combine fixed and percentage rents. In this context, sales are a basis for tenant replacements, but the choice of rules represents an issue. The objective function for risk and return should be optimized for the rules on tenant combinations and replacement, but in the simulations in this paper, we compare rules based on the change in sales and those based on the level of sales, and propose the latter type of rule from the perspective of risk and return.

## Formulation of An Analytical Framework

In this section we formulate an analytical framework for treating tenant management issues raised in Sections 1 and 2. Throughout this paper a tenant manager is required to find an optimal strategy with use of the real options: percentage rent linked to sales and tenant replacement, which were made available by the new house lease law (NHLL).

We assume the lease agreements are for three years and, for simplicity sake, also assume that they include provisions that prohibit tenants from getting out of their leases before the term is up. The property management companies have the right to ask tenants to leave at the end of the leases.

The time frame for our valuation analysis is 30 years, broken down into 10 three-year contract periods ( $k=1,2,\dots, 10$ ). The current time period is denoted 0, and we derive the probability distribution of income capitalization values using DDCF for each monthly period ( $n=1,2,\dots, 360$ ). In the second contract period, for instance, the months that are analyzed are  $n=37,38,\dots, 72$ . To annualize the time frame, we set  $h=1/12$ . For instance, the time frame from month 0 to month  $n$  can be expressed in years as  $nh$ . The discount rate for future cash flows is expressed as an annualized rate.

The retail property has  $I$  spaces for lease,  $i=1,\dots, I$ . Each space is occupied by a tenant with a specific type of business. For simplicity it is assumed that a tenant is found on vacancy. The method proposed by Kariya, Ohara and



Honkawa (2002) can be applied to the case where there are stochastic vacancy periods after tenants leave.

### *Structure of the lease agreements*

We assume lease agreements for each contract period that combine fixed and percentage rents. The percentage-rent portion is based on monthly sales, and is assumed to be paid at the end of each month. Specifically, the per-3.3 m<sup>2</sup> rent for the  $i$ -th space (for a specific type of business) and the  $n$ -th month can be expressed as

$$X_{i,n}(k) \equiv X_i(k, m(k)), n = 36(k-1) + m(k) : 1 \leq m(k) \leq 36$$

This rent can be further expressed as

$$X_{i,n}(k) = (1 - \alpha_i) \tilde{X}_i^f(k) + \alpha_i \tilde{S}_{i,n}(k) \quad (1)$$

The left-hand side represents the rent received for the  $i$ -th space and the  $n$ th month of the  $k$ -th contract period. On the right-hand side,  $(1 - \alpha_i)$  represents the proportion of the rent for the  $k$ -th contract period that is fixed and  $\alpha_i$  represents the proportion of the rent that is tied to sales  $\tilde{S}_{i,n}$ . Hence, in Eq. (1), the rent is expressed as a sum of a fixed rent for the  $k$ -th contract period,  $\tilde{X}_i^f(k)$ , and a rent that is tied to sales in the  $n$ th month,  $\tilde{S}_{i,n} \equiv \tilde{S}_i(k, m(k))$ . Contract sales  $\tilde{S}_{i,n} \equiv \tilde{S}_{i,n}(k)$  are defined in the lease agreement to reflect the differing levels of sales for different types of businesses and the actual variability of tenants' sales  $S_{i,n}(k)$ . We provide the formulation later on. The initially determined fixed rent for the contract term is expressed as  $\tilde{X}_i^f(k) = \tilde{X}_{i,12(k-1)}$  where  $\tilde{X}_{i,n} \equiv \tilde{X}_i(k, m(k))$  is the market rent at time  $n$ , with  $n = 36(k-1) + m(k)$ . It is noted that  $\alpha_i$  is a control parameter (variable) to be chosen for an optimal strategy in the environment of uncertainty about market rents and tenant's sales.

The dependence on the type of business  $i$  with parameter  $\{\alpha_i : i = 1, \dots, I\}$  in Eq. (1) is related to variability of sales. When  $\alpha_i = 0$ , i.e.,

$$X_{i,n}(k) = \tilde{X}_i^f(k) = \tilde{X}_{i,36(k-1)} \quad (2)$$

the agreement is a typical fixed-rent one, with each fixed rent constant for

three years determined at the time of contract.

We first take the case of  $k = 1$  to consider contract sales  $\tilde{S}_{i,n}(k)$  and percentage rents based on the contract sales. The rent at time  $n = 1$  (the end of the first month) is

$$\begin{aligned} X_{i,1}(1) &= (1 - \alpha_i)\tilde{X}_i^f(1) + \alpha_i\tilde{S}_{i,1}(1) \\ \tilde{X}_i^f(1) &= \tilde{X}_{i,0} \end{aligned} \tag{3}$$

$\tilde{X}_{i,0}$  is the market rent at time 0 adjusted for the type of business, and is fixed for 36 months.  $\tilde{S}_{i,1}(1)$  is a random variable at time  $n = 1$ . The term  $X_{i,1}(1)$  on the left-hand side is also a random variable at time  $n = 1$ . If we denote sales of a tenant at time  $n$  as  $S_{i,n}$ , and the annual variation in sales between time  $n-1$  and  $n$  as

$$r_{i,n} = \frac{1}{h} \log(S_{i,n}/S_{i,n-1}), \quad h = 1/12 \tag{4}$$

then the actual level of sales can be expressed as the identity

$$S_{i,n} = S_{i,n-1} \exp(r_{i,n} h) \tag{5}$$

When  $n = 1$ ,  $S_{i,n}$  is observable, but  $S_{i,0}$  is not, and hence neither is  $r_{i,n}$ . However, when  $n \geq 2$  then  $r_{i,n}$  is observable. Management's required amount of sales at  $n = 1$  is

$$\tilde{S}_{i,1} = \tilde{X}_i^f(1) = \tilde{X}_{i,0} \tag{6}$$

and the initial rent after the tenant takes occupancy at time 0 is

$$X_{i,1}(1) = \tilde{X}_{i,0} \tag{7}$$

This amount is the required rent for the first month at time 0. The important point in this expression is that the management company's required rent for the first month, even when tied 100% to sales with  $\alpha_i = 1$  is nothing more than the fixed rent  $\tilde{X}_{i,0}$ . In this sense, contract sales are rationally indexed to actual sales.

Contract sales when  $n = 2$  with observable  $r_{i,2}$  are defined as

$$\tilde{S}_{i,2} = \tilde{S}_{i,1} \exp(r_{i,2} h) = \tilde{X}_{i,0} \exp(r_{i,2} h) \tag{8}$$

The rent in this case is  $X_{i,2}(1) = (1 - \alpha_i) \tilde{X}_i^f(1) + \alpha_i \tilde{S}_{i,2}$ . Similarly, if contract sales with observable  $r_{i,n}$  are defined as  $\tilde{S}_{i,n} = \tilde{S}_{i,n-1} \exp(r_{i,n} h)$ , then the rent at time  $n$  with  $36 \geq n \geq 3$  is given by

$$X_{i,n}(1) = (1 - \alpha_i) \tilde{X}_i^f(1) + \alpha_i \tilde{S}_{i,n} \tag{9}$$

This is nothing more than Eq. (1) with  $k = 1$ .

For the  $k$ -th contract period as well, the rent for a tenant that continues to lease space and is not asked to vacate starting in the  $(k-1)$ -th contract period is given by Eq. (1). But in the case of a new tenant that takes occupancy at the end of  $n = 36(k-1)$ , the fixed-rent portion is the market rent at time  $36(k-1)$ , i.e.,  $\tilde{X}_i^f(k) = \tilde{X}_{i,36(k-1)}$ . The percentage rent at time  $36(k-1) + 1$  is based on contract sales, as in Eq. (6), with  $\tilde{S}_{i,12(k-1)+1} = \tilde{X}_i^f(k)$  and

$$X_{i,36(k-1)+1}(k) = \tilde{X}_{i,36(k-1)} \tag{10}$$

From time  $n \geq 36(k-1) + 2$  onward, the rent is defined in Eq. (1) based on contract sales as expressed in Eq. (8).

*Tenant-replacement rules*

The importance of tenant management lies in increasing the DDCF value of the property by putting in tenants with the ability to attract customers so that the tenants benefit mutually from externalities. One way to do so is to actively replace tenants. A practical indicator for the ability to attract and retain shoppers is sales. We express a tenant-replacement rule for the end of a contract period based on sales for that period as

$$F(S_{i,36(k-1)+1}, \dots, S_{i,36k}) \geq 0, \quad k = 1, \dots, K \tag{11}$$

Specifically, the rule that the average change in sales in the past two years, through six months prior to the end of the  $k$ -th contract period (which factors in the tenant's vacancy preparation period and seasonal variations in sales), can be expressed as

$$\bar{r}_i(k) = \frac{1}{24} \sum_{j=36(k-1)+7}^{36k-6} r_{ij} \geq c(k) \tag{12}$$

When the change in sales under such a lease agreement is negative, it is

rational to demand that the average profitability be at least a certain level, since the rent could be below the required fixed rent. We call this rule the average sales growth tenant replacement (ASGTR) rule.

Another possible rule is that the amount of contract sales six months prior to the end of the contract period be at least a certain level, as follows:

$$\tilde{S}_{i,36k-6}(k) = \tilde{S}_{i,36(k-1)+1} \exp\left(\sum_{j=36(k-1)+2}^{36k-6} r_{ij} h\right) \geq c(k) \tag{13}$$

We call this the sales level tenant replacement (SLTR) rule.

These two rules are considered in this paper though one may consider a third one. Note that a choice of the threshold  $c(k)$  in Formulae (12) and (13) for each rule leads to a different performance in the expected value and downside risk of the DDCF property distribution. That is,  $c(k)$  in each rule is a control parameter to be chosen for an optimal strategy and assumed to be a constant,  $c(k)=c$ , in our analysis.

*DDCF value and its distribution*

Given the above lease agreement structure and tenant-replacement rules, the DDCF value of future cash flows is

$$V_i = \sum_{k=1}^K V_i(k) \tag{14}$$

This is a stochastic variable, following the DDCF property distribution. Here the DDCF value of future cash flows from the  $i$ -th tenant for the  $k$ -th contract period is

$$V_i(k) = \sum_{n=n_{i,k-1}+1}^{n_i} \left[ (1-\alpha_i) \tilde{X}_i^f(k) + \alpha_i \tilde{U}_{i,n}(k) \right] A_i D(n) \tag{15}$$

where  $D(n)$  is the discount rate for cash flows at time  $n$  and  $A_i$  is the size of space  $i$  in 3.3 m<sup>2</sup>, and  $\tilde{U}_{in}(k)$  represents the tenant’s sales for the  $k$ -th period, and as noted earlier regarding the first contract,

$$\tilde{U}_{i,n}(1) \equiv \tilde{S}_{i,n}(1) \tag{16}$$

For the second contract period onward, a change in tenants is a possibility, and so to distinguish between tenants that stay and those that leave, we designate the following:

$$L_i(k) = \begin{cases} 1 & \text{if } F_k(\tilde{S}_{36(k-1)+1}, \dots, \tilde{S}_{36k}) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

Based on this function, contract sales for the  $i$ -th tenant for the  $k$ -th contract period are

$$\tilde{U}_{i,n}(k) = \tilde{U}_{i,n}(k-1)L_i(k-1) + \tilde{S}_{i,n}(k)[1 - L_i(k-1)] \quad (18)$$

When  $L_i(k) = 1$ , this equation expresses the sales of a tenant that stays from the  $(k - 1)$ -th contract period. In other words, whereas  $n > 36(k - 1)$ ,  $\tilde{U}_{i,n}(k - 1)$  relates to percentage rents that are extended from the  $k$ -th period, given that the agreement is extended from the  $(k - 1)$ -th period to the  $k$ -th period.

The distribution of  $V_i$  in Eq. (14) cannot be derived analytically for a given tenant replacement rule even if the models of the market rent process and the sales process are rather simple. However, via Monte Carlo simulation it can be derived numerically with sufficiently many sample paths. In Section 5 we carry out the derivation for the ASGTR and SLTR rules in Formulae (12) and (13). From this distribution we obtain such summary statistics as mean, standard deviation, semi-deviation, minimum, maximum and quantiles of a DDCF distribution for given  $\alpha_i$  and tenant-replacement rule.

### Optimization problem

When one of the tenant-replacement rules in Formulae (12) and (13) is used, the DDCF distribution of  $V$  in Eq. (14) is dependent nonlinearly on the control parameter  $(\alpha, c)$  that we call strategy, where the suffix  $i$  is deleted for simplicity. In fact, in our framework  $(\alpha, c)$  forms a strategy for controlling the shape of the DDCF distribution to enhance value creation where  $\alpha$  is the parameter for percentage rent linked to sales and  $c$  is the threshold parameter in one of the tenant replacement rules Formulae (12) and (13). To describe our analysis in the sequel, let  $M$  and  $R$  be respectively the mean (expected) value and the lower semi-deviation of the DDCF distribution of  $V$  in Eq. (14), which we adopt as return and risk respectively. Here the lower semi-deviation is defined as  $\{E[(\min(V - M, 0))^2]\}^{1/2}$ , which measures a downside risk that the DDCF value  $V$  falls in the interval  $[0, M]$ .

Now, since a change in strategy  $(\alpha, c)$  changes the DDCF distribution of  $V$ ,  $M$  and  $R$  are nonlinear functions of  $(\alpha, c)$ :

$$M = M(\alpha, c), \quad R = R(\alpha, c) \quad (19)$$

Thus an optimal strategy  $(\alpha^*, c^*)$  may be formalized as a strategy optimizing an objective function, say,  $G=G(M(\alpha, c), R(\alpha, c))$ , though the optimization problem is not analytically solvable because of the nonlinearity. As such an objective function, one may use the return-risk ratio  $M(\alpha, c)/R(\alpha, c)$ , return per unit risk. In the Markowitz portfolio theory this scheme is commonly adopted though the return and squared risk in the portfolio theory are respectively linear and quadratic functions of the control parameters, which does not hold in our case. In Section 7 the set of strategies  $\{(\alpha, c)\}$  each of which maximizes return  $M$  for each fixed risk  $R$  is numerically plotted. Along the portfolio theory, we call the set the efficient frontier, However, we are more interested in the values of the real options made available by the NHLL of 2000 and hence we derive an optimal strategy  $(\alpha^*, c^*)$  relative to the case of the OHLL where there are neither percentage rent nor tenant replacement.

It is remarked that a retail owner may sell a one-term renewal option which allows a new tenant to renew his lease contract for the second term for take-off when each lease term is short. Such an option is valued in our framework.

### Formulation of Our Model

An actual analysis using the framework provided in Section 3 requires:

- 1) a model for market rents, since the fixed rent determined at the start of each contract period is the market rate at that time; and
- 2) a model for the variability of sales for each type of tenant business.

We formulate these models below.

#### *A model for market rents*

The following log DD process is used as our market rent model (Kariya and Liu, 2002):

$$\tilde{X}_{i,n} = \tilde{X}_{i,n-1} \exp \left[ \mu_{X_{i,n-1}} h + \gamma_{X_{i,n-1}} \sqrt{h} \tilde{\varepsilon}_{X_{i,n}} \right] \quad (20)$$

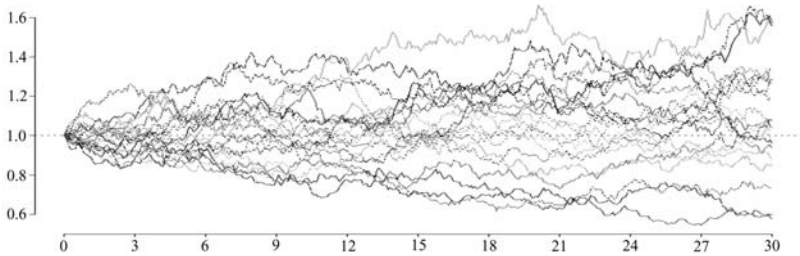
where drift  $\mu_{X_{i,n-1}}$  and volatility  $\gamma_{X_{i,n-1}}$  may depend on past values of  $\tilde{X}_{i,n}$  and  $\tilde{\varepsilon}_{X_{i,n}} \sim iid N(0,1)$ , the standard normal distribution. For the drift  $\mu_{X_{i,n-1}}$  for market rents, we use an exponential smoothing model, which is non-Markovian,

$$\mu_{X_{i,n-1}} = \phi_{X_i} \log \left[ \frac{\tilde{X}_{i,n-1}}{\tilde{X}_{i,n-2}} \right] + (1 - \phi_{X_i}) \mu_{X_{i,n-2}} \tag{21}$$

The model may depend on the tenant's type of business  $i$ , which we omit below.

This smoothing parameter  $\phi_X$  indicates the extent to which new information on rent changes  $\log[\tilde{X}_{n-1}/\tilde{X}_{n-2}]$  is discounted and reflected in the next rent level  $\tilde{X}_n$ . A small smoothing parameter  $\phi_X$  means that the monthly changes are slowly incorporated into a market trend movement. To express the dependence on the type of business, we use  $\tilde{X}_{i,n} = \lambda_i \tilde{X}_n$ . The rent is adjusted for the type of business by  $\lambda_i$ , and  $\tilde{X}_n$  represents the level of market rent. Figure 1 shows the sample path for the parameters in our base case.

**Figure1: Sample paths**



Note: 50 sample paths generated with the following diameters: initial drift  $\mu_{X_0} = 0\%$ , volatility  $\gamma_x = 5\%$ , initial market rent  $X_0 = 1$  (¥/3.3m<sup>2</sup>), and smoothing parameter  $\phi_x = 0.2$ .

*A model for the variability of sales*

The contract sales process defined in Eq. (9) is modeled based on a rate of return  $r_{i,n}$  with

$$r_{i,n} = \mu_{i,n-1} h + \gamma_i \sqrt{h} \varepsilon_{i,n}$$

Accordingly, the log DD model for the contract sales process in Eq. (9) is

$$\tilde{S}_{i,n}(k) = \tilde{S}_{i,n-1}(k) \exp\left[\mu_{i,n-1}h + \gamma_i \sqrt{h} \tilde{\varepsilon}_{i,n}\right] \quad (22)$$

where  $\mu_{i,n-1}$  and  $\gamma_i$  depend on past values of  $\tilde{S}_{i,n}$  and  $\tilde{\varepsilon}_{X_{i,n}} \sim iid N(0,1)$ . Here, as in the case of the model for rent variation, the drift  $\mu_{i,n}$  is described by an exponential smoothing model:

$$\mu_{i,n-1} = \phi_i \log \left[ \frac{\tilde{S}_{i,n-1}(k)}{\tilde{S}_{i,n-2}(k)} \right] + (1 - \phi_i) \mu_{i,n-2} = \phi_i r_{i,n-1} h + (1 - \phi_i) \mu_{i,n-2} \quad (23)$$

Volatility  $\gamma_i$  is assumed to be a constant. The volatility of sales is set to be greater than the volatility of market rents.

#### *The discount rate*

$D(n)$  represents the present value of ¥1 at time  $n$  in the future, which can be expressed as

$$D(n) = (1 + r(n))^{-nh} \quad (24)$$

where  $r(n)$  is the spot rate (annualized) for the period  $nh$  determined by the term structure of interest rates. From the perspective of the arbitrage pricing theory, it is natural to use a discount rate based on spot rates given by the term structure of interest rates, which is given at 0. In this case, the discount rate differs depending on the timing of the cash flows.

The discount rate used in the traditional static DCF valuation model is the exogenous cap rate  $r^*$  (a constant regardless of the timing of the cash flows) that reflects the complex risks associated with the uncertain profitability of real estate investments and thus includes a risk premium in addition to the risk-free rate.

Accordingly, in the case of an exogenous cap rate, a frequent subject of debate is how the cap rate is determined. There should be a variety of expected values, given that there are a variety of investors using a variety of cap rates.

In our DDCF perspective, even in the case of a flat term structure of interest rates as  $r(n) = r^*$ , we use the risk-free rate with no risk premium. For risk is derived directly from the probability distribution of DDCF values. In this paper, we use a constant term structure of interest rates and continuously



compounded rates.

## Valuation Using Monte Carlo Simulation

In this section various Monte Carlo simulations are carried out to derive the distributions of DDCF values of a retail property and find an optimal strategy for tenant management. In this section no consideration for tenant replacement cost is made. The case with the cost is treated in Sections 6 and 7. In this section we use Formulae (12) and (13) as tenant-replacement rules. Then in our framework  $(\alpha, c)$  forms a strategy where  $\alpha$  is the parameter for percentage rent linked to sales and  $c$  is the threshold parameter in one of Formulae (12) and (13). Using the notation in Eq. (19), let  $M = M(\alpha, c)$  and  $R = R(\alpha, c)$  be respectively the expected value and the lower semi-deviation of the DDCF distribution, which are regarded as return and risk respectively. In the Monte Carlo simulation the lower semi-deviation is computed as

$$\text{lower semi-deviation} = \sqrt{\sum_{n=1}^N \{\min(V_n - \bar{V}, 0)\}^2 / (N - 1)} \quad (25)$$

where  $V_n$  is the  $n$ -th DDCF value of the  $N$  DDCF values under each scheme and  $\bar{V}$  is the mean.

In the context of policy implication the case of fixed rent and no tenant replacement in the OHLL correspond to the case with  $\alpha=0$  and  $c = -\infty$  when the ASGTR rule (Formula (12)) is used and to the case with  $\alpha=0$  and  $c=0$  when the SLTR rule (Formula (13)) is used. For example, the case with  $\alpha=0$  and  $c = -\infty$  in Formula (12) means fixed rent and no tenant replacement because Formula (12) is always satisfied.

Now let  $\tilde{X}_{i,n} = \lambda_i \tilde{X}_n$ , where  $\lambda_i$  represents a rent adjustment for the type of business,  $\tilde{X}_n$  is the market rent, and the process is as described by the log DD process in Eq. (20). Below, we consider the case in which  $\lambda_i = 1$ . As a base case, we set the initial drift, volatility, smoothing parameter and initial market rent for the market rent process as

$$\mu_{X_0} = 0\%, \gamma_X = 5\%, \phi_X = 0.2, \text{ and } X_0 = 1 \text{ (¥/3.3m}^2\text{)} \quad (26a)$$

Unless otherwise noted, we use these base-case parameters. For the sales process in Eq. (22), we use the following parameters with the variation in sales that is greater than the variation in rent:

$$\mu = 0, \gamma = 0.2, \text{ and } \phi = 0.2 \quad (26b)$$

We also simply outline the case in which sales volatility is 10%.

The expected value of a retail space is the product of the mean of the distribution and the size of the space. Our analysis here does not address the issue of choosing between risk and return in the case of multiple tenants (tenant portfolio).

*Combinations of tenant-replacement rules and lease agreement structures*

In the following simulation, we consider lease agreement structures and tenant management in terms of:

- Case A: A contract with a fixed rent only ( $\alpha = 0$ ) and no replacement of tenants (base case)
- Case B: The inclusion of a percentage rent ( $\alpha > 0$ ), but no replacement of tenants
- Case C: A percentage rent and 100% replacement of tenants each period
- Case D: The inclusion of a percentage rent ( $\alpha > 0$ ), with the following types of rules for tenant replacements:
  - (I) ASGTR (Average sales growth) rule in Formula (12) and
  - (II) SLTR (Sales level tenant replacement) rule in Formula (13);
- Case E: The same case as Case D but with sales volatility of 10%.

Each distribution is based on  $N=100,000$  paths, each of which gives a DDCF value.

*Measures for valuing the real options*

Case A corresponds to the case of the OHLL, and all the other cases are based on the NHLL of 2000. The measures for valuing the real option obtained as a result of the transition from the OHLL to the NHLL are based on the return  $M$  and lower semi-deviation (risk)  $R$  of the distribution. Among others we use the measures

$$A = (M_N - M_O)/M_O, B = (R_O - R_N)/R_O, \text{ and } C = (M_N/R_N) - (M_O/R_O) \quad (27)$$

which are respectively the measures of return improvement, risk improvement, and return-risk improvement over the case of the OHLL. Here the suffixes N and O denote the cases of NHLL and OHLL respectively. Of course we are interested in the set of strategies  $(\alpha, c)$  that derive positive values of all the three measures and an optimal strategy is the one that maximizes  $M$  given  $R$ . Note that the ratio  $M/R$  is the expected DDCF value per unit risk. If this is large, one may choose such a strategy even if risk is large. However, in our policy evaluation we focus on the case where all the three measures are positive, i.e., on the case of lower risk and higher return.

**Case A: Fixed rent and no replacement of tenants (OHLL)**

With  $\alpha_i = 0$ , i.e., no percentage rent and a fixed rent only, we analyze the DDCF distribution. The uncertainty in deriving the DDCF distribution for the property is the risk of variation in market rents, which determine the initial rent for each contract period.

The basic statistics of the DDCF distribution for the base case Formula (26a) are as follows:

Average, 317.0; Standard deviation, 49.04; Lower semi-deviation, 31.74; Minimum, 173.1; Bottom 5% quantile, 245.4; Bottom 10% quantile, 258.4  
(28a)

These numbers are listed in the first line of the Table 1. For policy implication, as risk measure we adopt Eq. (25) with the mean  $\bar{V}$  replaced by 317.0 even when the cases in Cases B, C, D, and E are treated.

$$R(\alpha, c) = \sqrt{\sum_{n=1}^N \{\min(V_n - 317.0, 0)\}^2 / (N - 1)} \tag{28b}$$

In other words, the risk is considered the downside part below the OHLL mean 317.0 in each case. The lower semi-deviation with this replacement is called lower semi-deviation 2. In this definition a smaller lower semi-deviation 2 guarantees a smaller possibility that the DDCF values fall below 317.0. In other words, we are interested in the set  $\{(\alpha, c): M(\alpha, c) > 317.0, R(\alpha, c) < 31.74\}$  to value the real options in terms of the measures in Formula (27). Note that the return-risk ratio  $M_0/R_0$  (return per unit risk) in the base case (OHLL) is 9.99

**Table 1: Change in DDCF distribution with percentage rents**

$\alpha$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.0	317.0	49.04	31.83	31.74	0.615	3.734	173.1	245.4	258.4	312.3	381.1	404.5	680.3
0.1	329.7	59.72	35.61	28.00	1.880	19.170	176.1	251.1	264.5	322.0	402.1	432.4	1903.5
0.2	342.5	89.43	45.35	29.78	3.955	54.635	168.2	246.5	260.7	325.3	437.1	490.6	3545.7
0.3	355.2	125.29	57.92	33.79	4.811	71.185	160.4	237.9	252.8	326.4	482.0	563.8	5187.9
0.4	368.0	163.30	72.22	39.22	5.144	77.868	148.5	226.5	242.2	326.8	530.4	641.1	6830.2
0.5	380.8	202.25	87.56	45.64	5.289	80.804	131.8	212.8	229.7	327.0	580.8	720.4	8472.4
0.6	393.5	241.69	103.54	52.78	5.357	82.200	115.2	197.5	215.8	327.8	631.4	799.3	10114.6
0.7	406.3	281.41	119.92	60.41	5.390	82.894	98.5	181.0	201.0	328.6	683.5	879.1	11756.8
0.8	419.1	321.31	136.57	68.40	5.406	83.242	81.8	163.3	185.3	329.6	735.2	958.7	13399.1
0.9	431.8	361.32	153.40	76.63	5.414	83.408	65.1	145.1	169.4	331.2	787.5	1038.9	15041.3
1.0	444.6	401.42	170.36	85.04	5.416	83.474	41.3	126.2	153.2	332.7	839.9	1119.3	16683.5

*Sensitivity of the distributional characteristics to changes of the parameters in Eqs. (20) and (21)*

First, we consider changes in the DDCF value distribution corresponding to changes in the volatility  $\gamma_x$  of market rent process.

Table 2a and Figure 1a show the different DDCF distributions in response to changes in the volatility  $\gamma_x$ , which is most sensitive to the DDCF property values. Figure 1a shows the density function and the distribution function. From the table and figure, it is evident that changes in  $\gamma_x$  have a significant impact on the forms of DDCF distribution. Specifically, when  $\gamma_x$  increases, we observe the following.

- 1) The expected value of the distribution is relatively stable up to about 5%, but the standard deviation rises sharply.
- 2) The skewness and kurtosis increase, skewing the distribution to the right. For a change in rent of up to about 5% annualized, the distributions have somewhat fat tails, but are similar to symmetrical normal distributions.
- 3) As risk measures, the minimum, the bottom 5%, and the bottom 10% consistently decrease, and the risk increases substantially.
- 4) As evident from the graphs of the distribution function, the distribution changes at around 311.2 in response to changes in  $\gamma_x$ , and the probability below that is roughly 0.46.

In addition, the maximum and the upper percentage quantiles increase, making the structure a high-risk, high-return one.

**Table 2a: Dependence of property values on  $\gamma$**

$\gamma$ (%)	Average	Standard deviation	Semi-deviation 1	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
1	311.2	9.44	6.57	10.20	0.11	2.993	273.6	296.0	299.2	311.0	323.5	327.0	352.2
2	311.8	19.05	13.02	16.09	0.25	3.114	243.9	282.0	288.1	311.0	336.7	344.5	413.0
5	316.8	48.94	31.77	31.77	0.61	3.729	174.2	245.4	258.3	312.1	381.2	404.1	656.1
10	336.7	110.48	64.64	52.26	1.35	6.482	108.8	198.4	218.4	316.7	479.6	543.0	1382.5
20	429.9	352.22	154.15	78.90	4.92	79.396	61.4	139.7	164.9	330.7	790.8	1043.7	15736.9

Note: Property values corresponding to  $\gamma$  values of 1%, 2%, 5%, 10%, and 20%, with  $\varphi=0.2$ ,  $\mu_0=0\%$ , and  $X_0=1$ .

**Table 2b: Dependence of property values on  $\mu_0$**

$\mu$ (%)	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0	317.1	49.16	31.90	31.75	0.616	3.730	174.4	245.6	258.4	312.3	381.8	405.1	665.3
1	318.1	49.23	31.93	31.15	0.614	3.667	167.3	246.4	259.3	313.3	382.8	406.4	606.9
2	319.4	49.44	32.09	30.56	0.612	3.704	164.3	247.1	260.4	314.6	384.4	407.8	654.0
5	322.9	50.13	32.55	28.96	0.599	3.604	182.7	249.6	263.0	318.0	389.4	412.9	613.2
10	329.9	51.29	33.36	25.86	0.607	3.772	178.1	254.7	268.4	325.1	397.6	421.6	753.9
20	343.7	53.62	34.80	20.39	0.619	3.754	189.0	265.6	279.7	338.6	414.3	439.3	710.4
50	388.8	61.77	40.15	8.90	0.608	3.722	204.6	298.6	314.8	383.0	469.7	499.2	794.8

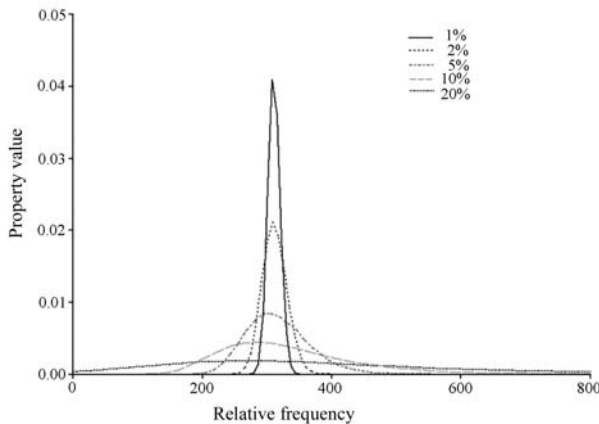
Note: Property values corresponding to  $\mu_0=0\sim 50\%$ , with  $\phi=0.2$ ,  $\gamma=5\%$ , and  $X_0=1$ .

**Table 2c: Dependence of property values on  $\phi$**

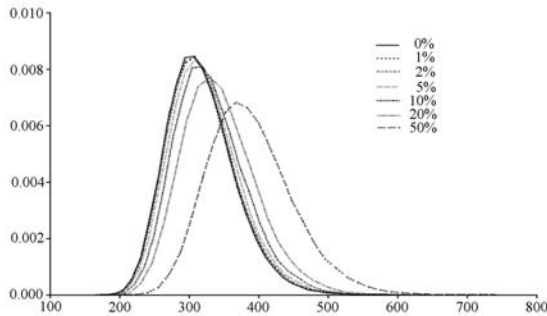
$\phi$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.0	317.1	49.18	31.90	31.77	0.616	3.730	174.4	245.5	258.4	312.3	381.8	405.1	665.3
0.2	317.8	53.83	34.69	34.05	0.664	3.801	168.6	240.7	254.5	312.4	388.1	414.0	709.4
0.4	318.2	53.98	34.74	33.81	0.671	3.781	170.6	241.2	254.7	312.7	389.1	414.5	663.9
0.5	318.3	54.04	34.86	33.86	0.653	3.763	174.9	240.8	254.5	312.8	389.6	415.2	699.0
0.6	318.2	54.11	34.83	33.94	0.683	3.930	165.3	241.1	254.5	312.6	389.1	414.7	769.1
0.8	318.2	54.01	34.81	33.88	0.667	3.841	166.5	240.9	254.5	312.8	389.0	414.7	677.3
1.0	318.2	54.09	34.84	33.93	0.673	3.865	167.6	240.9	254.4	312.7	389.0	414.3	694.3

Note: Property values corresponding to  $\phi$  values of 0, 0.2, 0.4, 0.5, 0.6, 0.8, and 1.0, with  $\gamma=5\%$ ,  $\mu_0=0\%$ , and  $X_0=1$ .

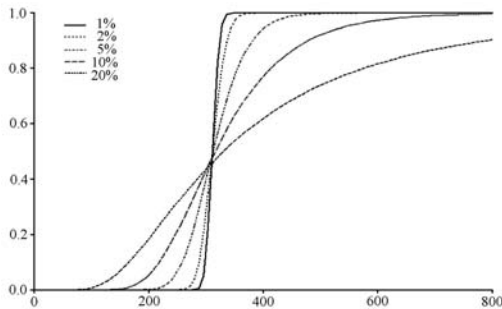
**Figure 1a: Dependence of property values on  $\gamma$**



**Figure 1b: Dependence of property values on  $\mu_0$**



**Figure 1c: Dependence of property values on  $\phi$**



Next, we consider changes in the initial drift  $\mu_0$  from the base case. As Table 2b shows, the distribution shifts to the right as  $\mu_0$  increases, and the standard deviation also increases, albeit slightly. Also, as  $\mu_0$  increases, the probability of larger DDCF values rises. This trend is related to the setting of  $\gamma_x = 0.05$ , but there is no indication of a significant change in the shape of the distribution. Also, the minimum and the lower percentage quantiles increase, and the risk decreases.

Finally, we consider changes in the drift smoothing parameter  $\phi_x$ . When  $\phi_x$  is 0, the drift stays at its initial value (0 in the base case), and as  $\phi_x$  approaches 1, the change in the drift becomes volatile. Perhaps because  $\gamma_x = 0.05$ , the shape of the distribution did not change that much, as shown in Table 2c and Figure 1c.

In sum, when the volatility of market rents is about 5%, the DDCF value distribution does not depend significantly on the values for  $\phi_x$  and  $\mu_{x_0}$ . It is roughly similar to a normal distribution, but in general terms it is not a normal distribution.

### **Case B: Mix of fixed and percentage rents, but no replacement of tenants**

Next, let us consider the case involving variable rents but no tenant replacement, which is a case of the NHLL. In other words, the sales process is similar to the market rent process, and the changes in the shape of the distribution in response to changes in the parameters are similar. Hence, we omit an analysis of this case, but present some results with percentage rents. This case, however, involves risk since tenants are not replaced, and can thus be considered an extension of cases based on the OHLL.

The base case for the market rent process is Eq. (20). The base case for the sales process is as in Eq. (22), with  $\mu = 0$ ,  $\varphi = 0.2$ , and  $\gamma = 0.2$ , and the assumption that sales variation risk is greater than market rent variation risk. The summary statistics of the DDCF property distribution in this case are given in Table 1, where the lower semi-deviation 2 in the table is defined as Eq. (28b) with the own mean in each case being replaced by the mean 317.0 of the OHLL case.

#### *Observations from Table 1*

We depart from the initial fixed rent toward the percentage rent with rate  $\alpha$ .

- 1) Compared with the case in which  $\alpha = 0$  (fixed rent only), an increase in  $\alpha$  leads to an increase in the mean and standard deviation, and this case thus shows a high-risk, high-return structure in a general sense.
- 2) However, among the key indicators for downside risk, the minimum and the bottom 5% and 10% quantiles all increase when  $\alpha = 0.1$  and  $0.2$ , compared with the case  $\alpha = 0$ , and hence the risk declines.
- 3) Similarly, the lower semi-deviation 2 declines, indicating a decline in the risk. Accordingly, in this case the property management company has a high possibility of increasing earnings by applying tenant-replacement rules.

Figure 2a illustrates sample paths of sales. The distributions for  $\alpha = 0.1, 0.2$  in Figure 2b is further to the right than in the case when  $\alpha = 0$ , and hence the probabilities are better (the DDCF distributions with  $\alpha = 0.1, 0.2$  are stochastically larger). In this sense, a 20% weight for percentage rent could be considered appropriate, even without tenant

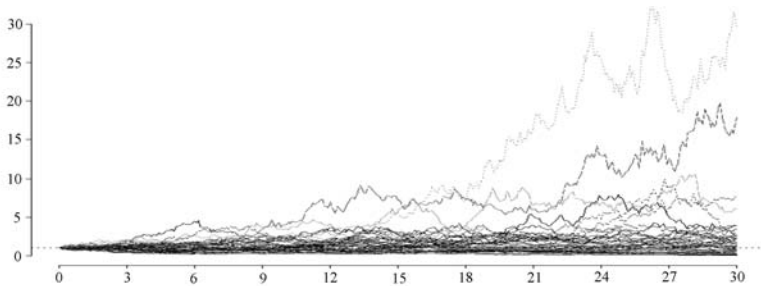
replacement, as long as the variation in sales is in line with our assumption. However, this assumes that a good choice of tenant is made in the first place.

When  $\alpha = 0.2$ , the additional contribution of the NHLL to the case of the OHLL is measured by Eqs. (27) with lower semi-deviation 2 as risk. The return improvement, risk improvement, and return-risk improvement are, respectively,

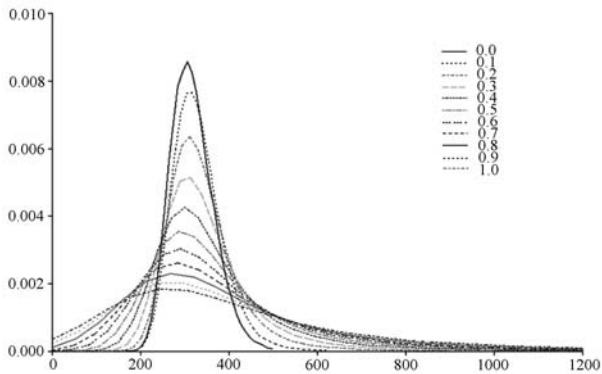
$$A = (342.5 - 317.0) / 317 = 0.08, \quad B = (31.74 - 29.78) / 31.74 = 0.062, \\ C = (342.5 / 29.78) - (317.0 / 31.74) = 1.51.$$

The values are not all that great, but they are encouraging because  $A$  and  $B$  are both positive, implying low risk and high return and there still remains a tenant replacement option.

**Figure 2a: Sample paths for sales**



**Figure 2b: Change in DDCF distribution with percentage rents**





**Case C: Mix of fixed and percentage rents and 100% replacement of tenants each period**

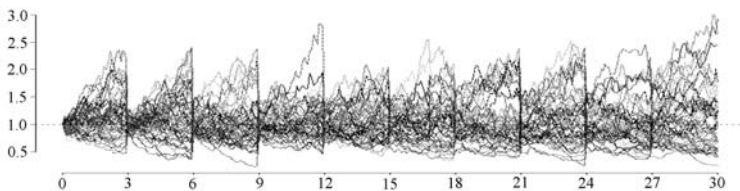
We next consider the use of percentage rents and active replacement of tenants. We provide our simulation results for the most extreme case, in which all tenants are replaced at the end of each contract period, with all other conditions the same as in the previous case.

Unlike in the previous case, when we increase the weight  $\alpha$  of the percentage rent, all the statistics are rather stable. Compared to the results in Table 1, the standard deviations in Table 3 are significantly smaller even though the mean and median increases, since the replacement of tenants at each contract period leads to a divergence in rent and sales paths and a reversion to original fixed-rent levels (see Figure 3). Table 3 shows that as  $\alpha$  increases, the mean and median increase slightly, and risk declines slightly since the minimum and the bottom 5% and 10% quantiles all increase slightly. In this sense, this case is a noteworthy one. This is shown in Figure 3.

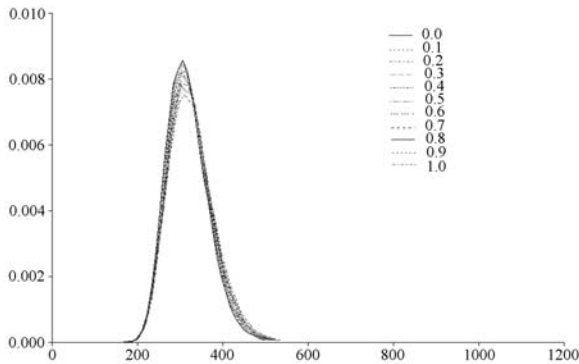
**Table 3: Change in DDCF distribution with percentage rents**

$\alpha$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.0	317.0	49.04	31.83	31.74	0.615	3.734	173.1	245.4	258.4	312.3	381.1	404.5	680.3
0.1	318.0	49.25	31.97	31.23	0.614	3.736	174.9	246.1	259.2	313.3	382.7	405.8	695.4
0.2	319.1	49.57	32.17	30.80	0.614	3.738	176.7	246.8	259.9	314.3	384.2	407.3	710.5
0.3	320.2	49.99	32.45	30.44	0.615	3.742	178.5	247.2	260.5	315.4	385.9	409.2	725.5
0.4	321.3	50.51	32.78	30.14	0.615	3.745	180.1	247.6	261.0	316.4	387.7	411.3	740.6
0.5	322.3	51.13	33.17	29.91	0.616	3.750	178.7	247.7	261.4	317.4	389.5	413.6	755.7
0.6	323.4	51.83	33.63	29.75	0.617	3.755	177.1	247.7	261.6	318.5	391.5	415.9	770.8
0.7	324.5	52.63	34.14	29.64	0.619	3.761	174.9	247.6	261.8	319.4	393.7	418.6	785.8
0.8	325.5	53.51	34.70	29.60	0.621	3.767	172.8	247.4	261.7	320.3	395.9	421.3	800.9
0.9	326.6	54.47	35.31	29.61	0.623	3.773	170.7	247.1	261.6	321.3	398.1	424.2	816.0
1.0	327.7	55.51	35.97	29.67	0.625	3.779	168.5	246.7	261.5	322.3	400.5	427.0	831.1

**Figure 3a: Sample paths at the time tenant replacement adopted**



**Figure 3b: Distribution of property values as a result of full tenant replacement**



In comparing Cases B and C, we can make the following point. When the OHLL does apply, the use of percentage rents is very risky as long as the original tenant stays. The increase in risk is greater than the increase in return. On the other hand, when the NHLL can be fully utilized and tenants are replaced each contract period, the return can be increased while holding down the risk. Accordingly, the use of tenant-replacement rules to select high-sales tenants can increase the average property value and at the same time minimize risk.

When  $\alpha = 0.5$ , the measures in Eqs. (27) for the additional real option values due to the NHLL with 100% replacement of tenants are  $A=0.0167$ ,  $B=0.058$ , and  $C=0.788$ . Though this case is better than the case of the OHLL with fixed rent and no replacement of tenants, which implies low risk and high return, these measures are worse than the case of the percentage rent only in Case B. This is because the tenant manager loses good tenants as well at each period by completely resetting.

**Case D: Mix of fixed and percentage rents and use of tenant-replacement rules**

In this subsection, we analyze cases of tenant management using tenant-replacement rules. Here we treat the case of  $\alpha = 0.5$ , i.e., 50% for percentage rents. A more comprehensive treatment is made in Section 7 together with tenant replacement cost. The parameters for the models are the same as in the previous section.

Specifically, we analyze how tenant management—based on two types of tenant-replacement rules, (I) average change in sales in Formula (12) and



When  $c = -0.4$ , the most relaxed threshold for tenant replacement in this analysis, the probability of contract extension at the end of each contract period is about 90%, as shown in Table 4b. As a result, the shape of the distribution is almost the same as the one in the case in which the ASGTR rule is not used (Table 1). As  $c$  increases and approaches 0 from below, the probability of contract extension declines. Accordingly, the standard deviation of the distribution also declines. By contrast, the mean and the median increase as  $c$  approaches  $-0.1$  (low risk, high return).

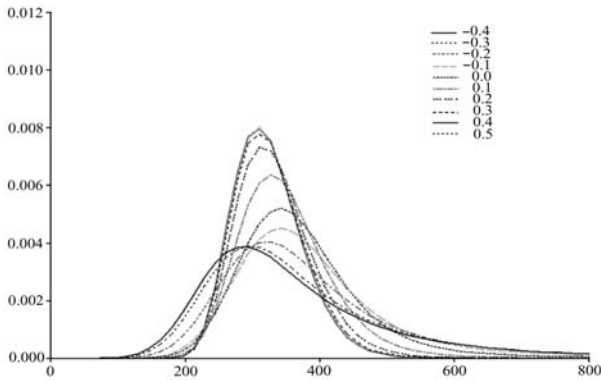
In this zone, the shape of the distribution narrows with the kurtosis and the minimum bigger and shifts overall to the right, a situation that approaches one preferable for a property management company seeking earnings with minimal risk. In fact, the mean and median greatly increase, while the minimum and the bottom 5% and 10% quantiles greatly increase with lower semi-deviations decreasing. This implies that an adoption of the ASGTR leads to both a great return enhancement and a great risk improvement. When  $c$  is greater than 0, the mean and the median decline. With the given parameters, an optimal case is when  $c=0$  or  $c = -0.1$ , either of which is an optimal strategy when  $\alpha = 0.5$ .

Given the sales process parameters and the ASGTR rule in this case, the contract extension probability is about 75% when  $c = -0.1$ , 50% when  $c = 0$ , and 25% when  $c = 0.1$ . In addition, as shown in Table 4b, there are no evident differences in contract extension probabilities when the contract is renewed for 10 three-year periods. In other words, even renewing tenants with a high ASG as of the end of the previous contract period have an ASG three years later that is about the same as that of new tenants. The reason is that it is difficult to sustain a high ASG for an extended period. In any case, when the proportion is 50% each period, it is probably not desirable to have a change in tenants.

Compared to the case of the OHLL, the value of the option with a 50% percentage rent contract and a tenant-replacement rule of  $c = -0.1$  is given by the measures in Eq. (27):

$$A = 0.264, B = 0.35, \text{ and } C = 9.49,$$

which are substantial improvement in both return and risk relative to Cases B and C. In addition, the risk is quite minimal, in terms of lower semi-deviation 2.

**Figure 4: DDCF distribution with ASGTR rule**

#### *Sales level tenant-replacement rule (SLTR rule)*

Table 5a shows the simulation results for the DDCF distribution in the case of a tenant-replacement rule Formula (13) based on the sales level of contract sales in two years and six months later, which we call the SLTR rule. The threshold value  $c$  for the rule in Formula (13) is to be chosen as an strategy with  $\alpha = 0.5$  given. In addition, Table 5b shows the contract extension probabilities at the end of each contract period.

As in the case of the sales growth replacement rule, the contract extension probabilities decline as the threshold becomes stricter ( $c$  increases) (Figure 5). However, unlike in the case of the ASGTR rule, the contract extension probabilities increase as time passes. It should be obvious that tenants that have strong business and have their contracts renewed have higher sales than new tenants do. When  $c=1$  and tenants are required to maintain the sales level at the time they took occupancy, about 50% of the tenants are able to satisfy this criterion at the end of the first contract period. This proportion increases steadily, to about 60% by the second contract period and to 77.6% by the ninth contract period.

The shape of the distribution changes as a result of changes in the contract extension probability related to changes in criteria. When  $c=1$ , the standard deviation of the distribution is large, but the distribution overall shifts to the right, while the downside risk diminishes. In fact, as  $c$  increases from below to 0.8, the mean and median substantially increase, the minimum and the bottom 5% and 10% quantiles increase substantially and the lower semi-deviation 2 (risk) decreases substantially, implying a great enhancement in both return and risk. The optimal threshold in this case will be in the interval (0.8, 1). The lower 5% and 10% quantiles are larger in the case of SLTR rule

with  $c = 1$  than in the case of ASGTR rule with  $c = -0.1$ . That the SLTR rule is better is also evident from the decline in the lower semi-deviation 2.

Compared with the results for the fixed-rent base case (Bin (28a)), the mean is much higher; the minimum and the values for the lower quantiles 5% and 10% are higher; the lower semi-deviation 2 is lower; and the downside risk is lower. In this sense, the value of the options stemming from the NHLL is very high as the measures in Eq. (27) are

$$A=0.38, B=0.43 \text{ and } C=14.1.$$

Compared to the case of the ASGTR rule with result  $A=0.26, B=0.35$ , and  $C=9.49$ , the improvement in both return and risk is much greater.

**Table 5a: DDCF Distribution with the SLTR rule**

$c$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.2	384.4	195.23	83.67	39.23	4.468	45.893	154.4	226.6	242.0	329.4	579.0	718.0	5607.3
0.4	399.2	190.68	80.15	27.92	4.649	49.015	173.9	248.5	263.2	345.2	585.1	724.3	5607.3
0.6	417.5	190.62	79.73	19.54	4.864	56.860	176.6	266.2	282.0	364.4	601.7	743.3	6394.1
0.8	433.6	193.52	81.95	16.29	4.792	55.416	176.6	275.6	293.7	381.2	621.5	762.2	6394.1
1.0	438.4	195.84	85.00	18.23	4.615	52.350	174.5	269.2	290.6	388.3	629.9	771.0	6394.1
1.2	427.1	194.15	85.30	22.37	4.637	53.864	174.5	257.6	276.9	380.1	616.0	753.6	6394.1
1.4	405.3	184.99	79.61	25.85	4.944	61.755	174.5	251.5	267.9	357.5	581.8	714.0	6394.1
1.6	381.1	167.65	68.79	28.01	5.681	82.054	174.5	249.1	263.8	336.2	535.5	654.5	6394.1
1.8	361.0	145.66	57.43	29.13	6.247	94.099	174.5	248.1	262.3	325.6	484.0	584.8	5607.3
2.0	347.0	124.42	48.76	29.67	7.037	124.495	174.5	247.7	261.7	320.8	441.2	524.1	5607.3

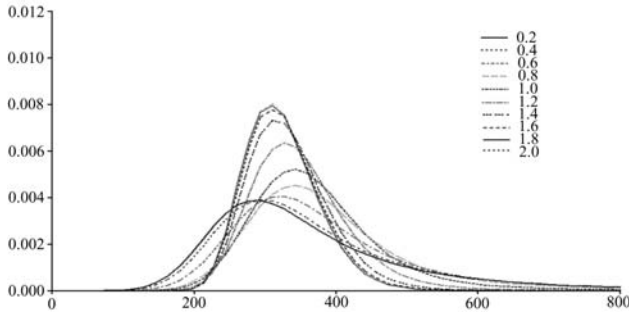
Note:  $c$  is annualized.

**Table 5b: Contract extension probabilities with the SLTR rule**

$c$	Three-year contract period								
	1	2	3	4	5	6	7	8	9
0.2	100.0	99.9	99.5	98.9	98.5	98.2	98.0	98.0	97.9
0.4	99.7	96.7	95.1	94.8	95.0	95.2	95.5	95.6	95.9
0.6	93.6	88.0	89.4	90.3	91.0	91.6	92.2	92.4	92.8
0.8	74.5	78.0	81.2	82.9	84.4	85.4	86.2	86.9	87.1
1.0	49.5	61.7	67.2	70.7	72.8	74.6	76.1	77.0	77.6
1.2	28.9	41.4	48.3	53.0	56.4	59.0	61.2	62.4	63.6
1.4	15.5	24.2	30.4	35.4	38.8	41.9	44.2	46.0	47.9
1.6	7.8	13.1	17.3	20.9	24.0	26.8	29.1	31.3	33.1
1.8	3.8	6.7	9.3	11.7	13.9	16.1	18.1	19.8	21.5
2.0	1.9	3.4	4.9	6.4	7.8	9.3	10.9	12.2	13.6

Note:  $c$  is annualized.

**Figure 5: Distribution of DDCF values with the SLTR rule**



**Case E: Sales volatility  $\gamma$  of 10%**

*ASGTR rule*

When sales volatility  $\gamma$  is 10%, Table 6 shows the characteristics of the DDCF value distribution with ASGTR rule, but we omit the graphs of the distribution. The return and downside risk are optimal when the threshold value  $c = -0.1$  or 0. Compared with the case in which  $c = 0$  and the fixed rent base case, the average is higher (337.2) and the minimum is lower, but the mean and the values for the lower percentage points are higher, resulting in lower downside risk overall. In addition, the contract extension probabilities are about the same as in the case when sales volatility is 20%, and tenant replacement occurs when the proportion is 50% though we omit the table.

**Table 6: Characteristics of value distribution with ASGTR rule**

$c$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
-0.4	327.5	64.55	39.12	32.72	1.135	5.719	163.0	241.8	255.9	317.9	411.0	446.0	921.4
-0.3	327.5	64.55	39.12	32.72	1.135	5.718	163.0	241.8	255.9	317.9	411.0	446.0	921.4
-0.2	328.4	64.31	38.98	32.08	1.136	5.733	163.0	242.9	257.0	318.9	411.6	446.5	921.4
-0.1	339.0	61.11	37.27	24.67	1.121	5.891	180.0	256.8	270.7	330.5	417.2	450.4	921.4
0.0	337.2	54.56	34.86	23.56	0.771	4.524	168.9	259.2	273.2	331.5	408.0	434.9	913.8
0.1	322.8	50.65	32.88	29.46	0.611	3.694	151.6	249.0	262.3	318.0	389.4	413.5	639.7
0.2	318.5	49.69	32.26	31.35	0.610	3.681	151.6	246.1	259.2	313.7	384.0	407.3	613.7
0.3	318.2	49.61	32.22	31.47	0.609	3.680	151.6	245.9	259.0	313.5	383.6	406.7	613.7
0.4	318.2	49.61	32.21	31.47	0.609	3.681	151.6	245.9	259.0	313.5	383.6	406.7	613.7
0.5	318.2	49.61	32.21	31.47	0.609	3.681	151.6	245.9	259.0	313.5	383.6	406.7	613.7

*SLTR rule*

On the other hand, the use of the SLTR rule corresponds to the case in which sales volatility is 20%. As shown in Table 7, overall optimization is achieved when the threshold value  $c = 0.8$  or 1, and results in a case that is better than the fixed-rent base case (Case C) as well as Case D(I), because of the higher return and lower downside risk. The contract extension probabilities are little different from those in Table 5b, and show a consistent increase, though we omit the table.

**Table 7: DDCF Distribution with SLTR rule**

$c$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.2	327.9	64.86	39.22	32.59	1.163	6.037	158.0	242.3	256.1	318.2	411.2	447.6	1189.6
0.4	330.0	63.26	37.72	29.80	1.250	6.351	174.2	248.4	261.0	319.5	411.4	447.8	1189.6
0.6	337.4	60.14	35.49	23.52	1.334	6.894	177.4	260.8	272.9	327.1	414.0	449.4	1189.6
0.8	348.4	59.26	35.83	19.03	1.207	6.718	175.3	269.8	284.2	339.8	423.0	456.5	1189.6
1.0	354.1	64.23	41.24	22.37	0.836	5.501	172.6	258.4	276.9	349.3	433.6	465.7	1189.6
1.2	343.5	69.02	43.81	28.48	0.842	4.739	172.6	248.8	262.6	336.3	432.1	464.3	1189.6
1.4	330.0	64.52	38.90	30.86	1.087	5.296	172.6	246.6	259.5	317.8	417.8	450.5	1029.0
1.6	323.1	58.17	35.07	31.29	1.144	5.798	172.6	246.2	259.1	314.2	399.1	433.7	1029.0
1.8	320.3	53.90	33.35	31.34	1.013	5.386	172.6	246.2	259.1	313.7	388.4	417.8	861.6
2.0	319.2	51.79	32.70	31.35	0.885	5.045	172.6	246.2	259.1	313.6	385.0	410.4	840.8

**Return-Risk Analysis With Tenant Replacement Costs**

Our discussion so far has ignored tenant replacement costs, which include space renovation costs and layout modification costs. The space renovation costs associated with tenant turnover  $\tilde{C}_i(k)$  can be expressed as a function of market rents at the time the contract is signed. We specify it as

$$\tilde{C}_i(k) = a + b\tilde{X}_{i,36(k-1)} \tag{29}$$

which is a linear function of market rents at the time the contract is signed. For instance, when  $a = 0$  and  $b = 12$ , the costs incurred by the property management company when there is a change in tenants are the equivalent of one year’s worth of market rent. In this formulation the mean and risk (lower semi-deviation 2 in Eq. (28b)) are respectively expressed as

$$M = M(\alpha, c; b) \text{ and } R = R(\alpha, c; b) \tag{30}$$



We consider the case  $\alpha = 0.5$  as a continuation of Case E. Hence for each tenant replacement rule and for each  $b$ , we are interested in finding an optimal threshold  $c$  in the set

$$\{(0.5, c); \tilde{M}(0.5, c; b) > 317.0, \tilde{R}(0.5, c; b) < 31.74\} \tag{31}$$

Table 8a shows the characteristics of the value distribution in the case of ASGTR rule and values of  $a = 0$  and  $b = 6$  for the cost function, i.e., costs equal to six months' worth of market rent. Table 8b shows the results for the same case but with a value of  $b = 12$ . Table 9 shows the results for the same pair of cases but with SLTR rule. Figures 6 and 7 show the graphs of  $\{(\tilde{R}(0.5, c; b), \tilde{M}(0.5, c; b))\}$  when  $c$  moves over its region for each rule and  $b=0,6,12,24,36$ .

Whichever rule is used, the higher the value of  $c$  and the greater the frequency of tenant turnover, the more the expected property value is affected by costs and declines, indicating greater downside risk than in the case in which tenant-replacement costs are ignored. The tables indicate that when the replacement costs are the equivalent of one year's worth of market rent, it is possible to increase the expected property value and minimize the downside risk by using tenant-replacement rules. Also, as Figure 6 shows, when the replacement costs reach the equivalent of two years' worth of market rent and the ASGTR rule is used, the expected property value effectively does not increase as a result of the adoption of the replacement rule. However, it is evident that when a SLTR rule is used, it is still possible to increase the expected property value, even when the replacement costs amount to three years' worth of market rent. This indicates that even when the replacement costs are high, the active replacement of tenants with weak sales leads to an increase in the property value.

**Table 8a: DDCF value distribution with ASGTR rule and  $a = 0, b = 6$**

$c$	Aver- age	Standard deviation	Semi- devi- ation	Semi- devi- ation 2	Skew- ness	Kurtosis	Min- imum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
-0.4	382.2	197.74	87.22	44.60	4.503	58.736	132.7	214.1	231.6	329.3	582.6	723.4	8295.8
-0.3	387.6	195.77	86.06	40.76	4.594	60.914	132.7	220.6	238.7	335.8	583.6	723.7	8295.8
-0.2	397.8	186.79	80.97	31.46	4.954	70.833	154.1	238.0	257.0	350.7	577.8	712.4	8295.8
-0.1	394.7	162.37	68.37	23.20	6.214	114.343	157.1	257.1	274.7	358.0	536.3	644.4	8295.8
0.0	364.9	112.52	52.24	24.71	5.743	89.926	165.1	254.6	271.1	343.8	470.9	532.8	4159.8
0.1	332.9	72.44	40.83	31.24	2.997	42.717	163.4	243.7	258.2	322.2	416.7	454.8	2588.4
0.2	312.5	56.14	34.86	37.59	1.130	8.171	161.7	235.0	248.4	305.8	384.0	412.2	1369.7
0.3	302.7	50.12	32.09	41.20	0.73	4.206	159.6	231.0	243.7	297.5	368.1	392.7	699.9
0.4	299.4	47.99	31.02	42.40	0.641	3.792	159.6	229.9	242.3	294.8	362.3	385.8	688.9
0.5	298.6	47.38	30.73	42.64	0.616	3.697	159.6	229.7	242.1	294.2	360.7	383.8	598.9

Note:  $c$  is annualized

**Table 8b: DDCF value distribution with ASGTR rule and  $a = 0, b = 12$**

$c$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
-0.4	382.2	197.75	87.22	44.64	4.503	58.728	132.7	214.0	231.5	329.2	582.6	723.4	8295.8
-0.3	387.1	195.85	86.06	41.00	4.593	60.853	132.7	220.3	238.2	335.1	583.2	723.7	8295.8
-0.2	395.7	187.16	81.06	32.45	4.944	70.468	150.6	236.2	255.0	348.1	576.4	711.3	8295.8
-0.1	388.7	163.32	68.77	26.09	6.165	112.461	152.1	250.6	268.3	351.3	532.2	641.4	8295.8
0.0	352.9	113.52	52.48	30.77	5.713	88.512	156.6	242.4	258.9	331.1	459.7	523.0	4159.8
0.1	315.0	72.15	40.13	41.28	3.133	44.705	154.1	227.7	241.8	303.9	397.9	436.6	2585.8
0.2	290.8	53.99	33.13	50.89	1.253	9.261	150.6	217.3	230.0	283.9	358.9	386.6	1355.5
0.3	279.4	46.72	29.74	56.13	0.782	4.457	147.7	213.1	224.7	274.3	340.4	363.4	673.0
0.4	275.7	44.11	28.46	57.83	0.658	3.857	147.7	212.0	223.3	271.3	333.5	355.1	666.2
0.5	274.8	43.38	28.11	58.17	0.620	3.707	147.7	211.8	223.1	270.7	331.7	352.9	558.6

Note:  $c$  is annualized.

**Table 9a: DDCF value distribution with SLTR rule and  $a = 0, b = 6$**

$c$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.2	362.5	197.29	83.44	52.02	4.901	58.794	145.6	206.4	221.0	306.7	557.0	694.8	6594.3
0.4	377.6	192.58	79.38	38.60	5.134	63.266	169.0	229.8	243.9	322.6	562.5	700.5	6594.3
0.6	396.6	190.95	78.28	27.28	5.177	64.092	180.8	250.2	264.7	342.6	578.5	716.6	6594.3
0.8	414.5	192.26	79.58	20.74	5.089	62.133	182.3	263.8	280.2	361.2	597.9	738.4	6594.3
1.0	422.3	193.44	81.31	20.31	5.012	60.960	176.2	263.5	283.1	370.9	606.6	746.1	6594.3
1.2	415.1	190.42	80.74	23.20	5.046	62.620	172.9	256.1	274.5	367.3	595.0	730.4	6594.3
1.4	397.0	179.80	75.23	26.10	5.301	70.888	172.9	251.5	267.1	351.2	563.8	689.4	6594.3
1.6	375.9	162.27	65.49	28.04	6.020	93.918	172.9	249.2	263.8	334.8	519.5	633.6	6594.3
1.8	358.2	142.33	55.44	29.07	7.262	140.323	172.9	248.2	262.1	325.2	472.5	572.1	6594.3
2.0	345.3	121.14	47.54	29.57	7.873	162.849	172.9	247.8	261.5	320.9	434.9	514.8	5955.3

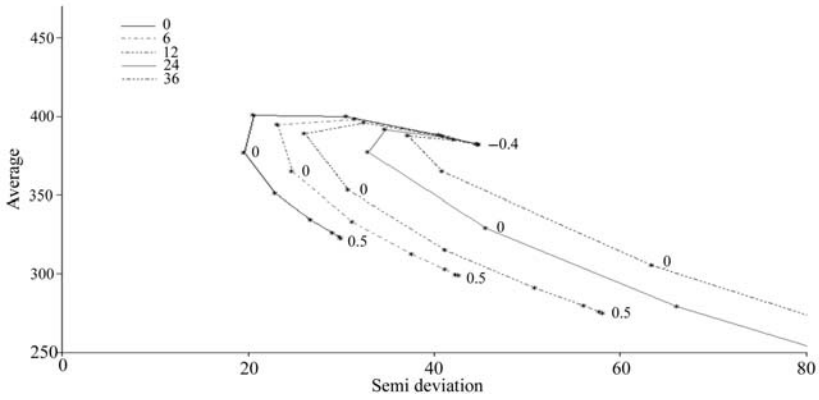
Note:  $c$  is annualized.

**Table 9b: Characteristics of value distribution with SLTR rule and  $a = 0, b = 12$**

$c$	Average	Standard deviation	Semi-deviation	Semi-deviation 2	Skewness	Kurtosis	Minimum	Bottom 5%	Bottom 10%	Median	Top 10%	Top 5%	Maximum
0.2	339.0	196.73	82.65	66.80	4.941	59.443	130.6	185.9	199.4	282.6	532.4	670.3	6570.1
0.4	354.7	191.70	78.20	51.52	5.197	64.351	157.6	211.1	223.9	298.8	538.1	675.8	6570.1
0.6	374.8	189.59	76.59	37.22	5.269	65.731	173.2	234.0	247.0	319.5	554.7	692.3	6570.1
0.8	394.6	190.12	77.05	26.87	5.226	64.564	179.6	252.0	266.0	339.9	574.5	714.3	6570.1
1.0	405.5	190.08	77.54	23.11	5.219	64.659	174.4	257.8	275.0	352.4	583.7	723.9	6570.1
1.2	402.1	185.74	76.00	24.34	5.329	67.980	172.9	254.4	271.6	353.5	573.2	708.0	6570.1
1.4	387.9	174.38	70.61	26.56	5.639	78.271	172.9	251.0	266.2	344.1	543.4	668.3	6570.1
1.6	370.1	156.96	61.90	28.21	6.405	104.450	172.9	249.1	263.5	332.0	501.5	613.6	6570.1
1.8	354.6	137.73	53.08	29.13	7.694	155.729	172.9	248.1	262.1	324.5	458.9	553.7	6570.1
2.0	343.2	117.39	46.12	29.59	8.263	178.982	172.9	247.8	261.5	320.6	427.9	500.5	5934.5

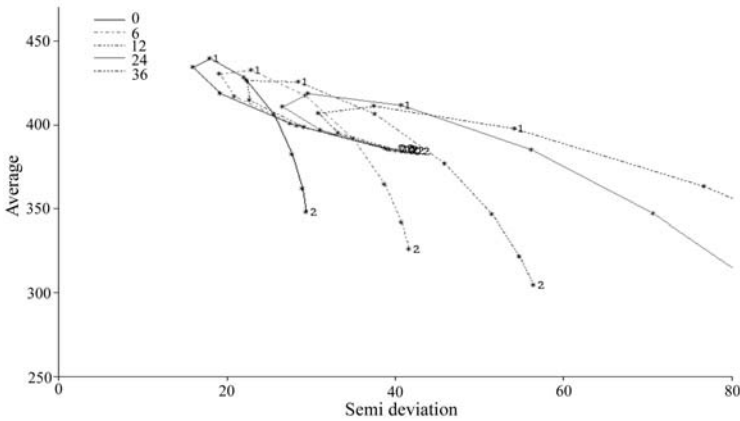
Note:  $c$  is annualized.

**Figure 6: DDCF expected property values and risk with ASGTR rule**



Notes:  $c$  is annualized; the graph shows plots of  $\{(R\tilde{0.5}, c; b), \tilde{M}(0.5, c; b)\}$  when the value of  $b$  in the replacement cost function is 0, 6, 12, 24, and 36 months; the points in the graph represent the replacement threshold value  $c$ .

**Figure 7: DDCF expected property values and risk with SLTR rule**



Notes:  $c$  is annualized; the graph shows plots of  $\{(R\tilde{0.5}, c; b), \tilde{M}(0.5, c; b)\}$  when the value of  $b$  in the replacement cost function is 0, 6, 12, 24, and 36 months; the points in the graph represent the replacement threshold value  $c$ .

### Efficient Frontier with Tenant-Replacement Cost Fixed

In Figures 6 and 7, the graphs of  $\{(\tilde{R}(0.5, c; b), M(0.5, c; b))\}$  when  $c$  moves over its region for each rule and  $b=0,6,12,24,36$  were drawn. Note that the percentage proportion of the variable rent and the threshold value  $c$  of the tenant replacement rules are control variables for a property manager, while the cost coefficient  $b$  is considered exogenous with  $a=0$ . In this section we set  $b=24$  and draw the graphs of  $\{(R(\alpha, c; 24), M(\alpha, c; 24))\}$  for feasible strategies  $\{(\alpha, c)\}$  and find an efficient frontier as in the Markowitz portfolio theory, through which we obtain an optimal  $\alpha$  for percentage rents for each given threshold  $c$ .

Figure 8 shows changes in the DDCF distribution as a result of changes in  $\alpha$  for percentage rents and replacement threshold  $c$ , with the use of the ASGTR rule. The solid lines in the figure show the graphs of  $(R(\alpha, c; 24), M(\alpha, c; 24))$  when the replacement threshold  $c$  is fixed but  $\alpha$  is changed. The dotted lines in the figure show the graphs of  $(R(\alpha, c; 24), M(\alpha, c; 24))$  when  $\alpha$  is fixed but  $c$  is changed.

There are combinations of possible tenant management strategies that can offer the same return with lower risk or higher returns with the same risk. For instance, the darker bold line in Figure 8 represents a frontier of such combinations of strategies when  $c = -0.2$  and  $\alpha$  changes from 0% to 100%. In this case,  $c = -0.2$  and  $\alpha = 20\%$  results in the maximum property value per unit of risk  $M_N/R_N = 346.78/26.68 = 12.96$  (see Eq. (27) for the notation). When the replacement cost is 24-month market rent, the return, risk, and return-risk improvements due to the NHLL over the case of the OHLL are

$$A=0.09, B=0.16, \text{ and } C=2.97.$$

Figure 9 shows the results for the same case but with the SLTR rule. In this case, a replacement threshold  $c$  of 0.6 and a 35% weight for the percentage rent results in the maximum property value per unit of risk  $M_N/R_N = 382.58/24.26 = 15.76$ . The average expected value is higher and the risk is lower than in the previous case (with the ASGTR rule). The improvements over the case of the OHLL are

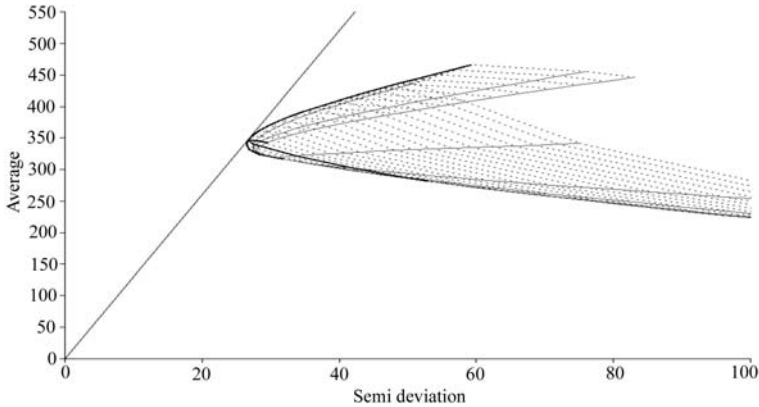
$$A=0.21, B=0.24, \text{ and } C=5.77.$$

Thus even if we take into the 24-month cost element, the improvements become relatively larger in terms of  $A$ ,  $B$ , and  $C$ . These results clearly show the effectiveness of the deregulation of the OHLL, providing real options to the managers and society.

The results of this section indicate that property managers can choose from among the possible tenant management strategies those that lead to optimal

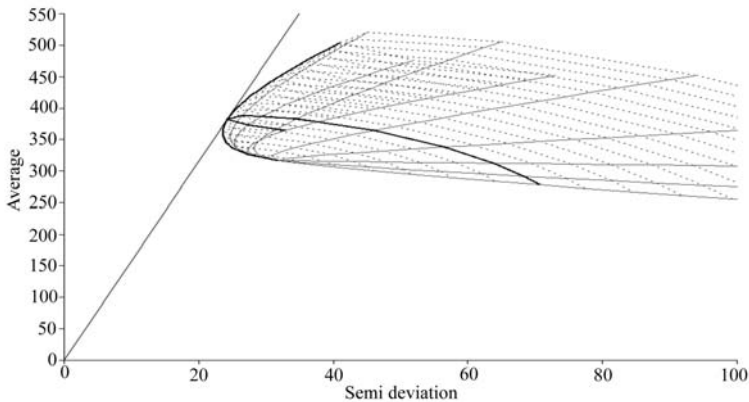
property value distributions for managers with a specific objective function.

**Figure 8: Combinations of tenant management strategies with the ASG-TR rule**



Note: The graph plots the combinations of  $(R, M)$  with  $b=24$ ; the solid lines show the graphs of  $(R, M)$  with  $c$  fixed but  $\alpha$  varying. The dotted lines in the figure show the graphs of  $(R, M)$  with  $c$  varying but  $\alpha$  fixed; the darker bold line represents the frontier of such combinations of strategies when  $c$  is set at  $-0.2$  and  $\alpha$  changes from  $0\%$  to  $100\%$ .

**Figure 9: Combinations of tenant management strategies with SLTR rule**



Note: The graph plots the combinations of  $(R, M)$  with  $b=24$ , the solid lines show the graphs of  $(R, M)$  with  $c$  fixed but  $\alpha$  varying. The dotted lines in the figure show the graphs of  $(R, M)$  with  $c$  varying but  $\alpha$  fixed; the darker bold line represents the frontier of such combinations of strategies when  $c$  is set at  $-0.2$  and  $\alpha$  changes from  $0\%$  to  $100\%$ .

## Conclusions

In this paper, we formulated the retail property management issue in terms of the structure of the tenant lease agreement and rules for replacing tenants, and proposed a new framework and methodology for assessing the expected property value and risk based on DDCF probability distributions for the property value stemming from different lease structures and tenant-replacement rules. Our simulation results based on the assumptions of our model indicate that active tenant management through the use of percentage rents and appropriate tenant-replacement rules changes the shape of the probability distributions of property values and results in the generation of value. In particular, we found that the optimal weighting for percentage rent can be derived by setting realistic parameters and using optimal tenant-replacement rules.

Our framework demonstrates the effectiveness of a tenant management strategy in creating corporate value. In particular, the results of our analysis showed the effectiveness of the tenant-replacement options made possible by the new House Lease Law and the options have substantial values additional to the value of the OHLL in return and risk.

## References

- Benjamin, J.D., Boyle, G.W. and Sirmans, C.F. (1990). Retail Leasing: The Determinants of Shopping Center Rents, *AREUEA Journal*, **18**(3), 302-312.
- Benjamin, J. D., Boyle, G. W. and Sirmans, C. F. (1992). Price Discrimination in Shopping Center Lease, *Journal of Urban Economics*, **32**, 299-317.
- Colwell, P. F. and Munneke, H. J. (1998). Percentage Leases and the Advantage of Regional Malls, *Journal of Real Estate Research*, **15**(3), 239-252.
- Chun, Eppli and Shilling. (1999). A Simulation Analysis of the Relationship between Retail Sales and Shopping Center Rents. Working paper.
- Grenadier, S. R. (1995). Valuing Lease Contracts: A Real-Options Approach, *Journal of Financial Economics*, **38**, 297-331.
- Grenadier, S. R. (1996). Leasing and Credit Risk. *Journal of Urban Economics*, **42**, 333-336.
- Kariya, T., Oohara, Y. and Honkawa ,H. (2002). A Dynamic Discounted Cash Flow Model for Valuation of an Office Building, Working paper

Kariya, T. and Liu, R. Y. (2002). *Asset Pricing Discrete Time Approach*, Kluwer Academic Publishers.

Miceli, T. J. and Sirmans, C. F. (1995). Contracting with Spatial Externalities and Agency Problems – The Case of Retail Leases. *Regional Science and Urban Economics*, **25**, 355-372.

Miceli, T. J. and C.F. Sirmans. (1995). Contracting with Spatial Externalities and Agency Problems: The Case of Retail Leases, *Regional Science and Urban Economics*, **25**, 355-372.

Wheaton, W C. and Raymond G. T. (1995). Retail Sales and Retail Real Estate, *Real Estate Finance*, **12**, 22-31.

Wheaton, W.C. (2000). Percentage Rent in Retail Leasing: The Alignment of Landlord-Tenant Interest. *Real Estate Economics*, **28**(2), 185-204.

.