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A Risk Management Model for MBS Issuers

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Most previous studies that developed Mortgage-backed Securities (MBS) models focused on investors, but the model that is presented here is specifically for MBS issuers. I developed a risk management tool for issuers and guarantors to monitor their MBS portfolios. The model projects the cash inflow of mortgages and the cash outflow to MBS; alters the traditional model by introducing decision trees; combines the prepayment, delinquency, default, and recovery of delinquency into a single model; and uses a simulation program with multiple path generation to develop a model for issuers to manage their MBS portfolios. According to the results of the model, issuers can manage the risk level of their portfolios by determining the Collection Account Balance, the Overcollateralization Ratio, the Net Residual Value, and the Liquidity Advance. The final part of this paper provides suggestions on risk management for MBS issuers.

Keywords

Mortgage-backed securities, risk management, MBS issuers, cash flow projection, Korea Mortgage Corporation

Introduction

The market for mortgage-backed securities (MBS) has experienced more rapid growth in size and complexity than any other fixed-income market in recent years. Since the earliest MBS transaction of the Government National Mortgage Association (Ginnie Mae) in early 1970s, various MBS have been issued by quasi-government agencies such as the Federal Home Loan Mortgage Corporation (Freddie Mac) and the Federal National Mortgage Association (Fannie Mae), or by private agencies and investment banks.

Nowadays, capital markets are looking for more complicated innovations in MBS products, such as collateralized mortgage obligation (CMO) and multiclass MBS. Numerous studies have been conducted into these complicated MBS, including those of prepayment analysis, delinquency management, and valuation models. Although countless studies have developed MBS, most of them have been written from the angle of MBS investors, and only a limited number have focused on MBS issuers.

The concerns of MBS issuers are completely different from those of MBS investors. The principal objective of MBS investors is to obtain the principal and interest return with a good balance between risk and reward expectations. As the cash flow of MBS depends on the prepayment rate of underlying mortgages, the amount and timing of monthly principal and interest payments is the focus of investors. In most situations, the concerns of investors arise from uncertain factors, such as the prepayment risk of mortgages, the structural risk of the MBS, and the credit risk of the issuers and guarantors, etc.

In contrast, the concerns of MBS issuers and guarantors are different from those of investors. Except for the plain vanilla with a simple pass-through structure, most MBS in current market are complicated CMOs with multi-class bonds. The principal and interest payments of CMOs are supported by the cash inflow of the underlying mortgages, and the cash outflows to investors are usually guaranteed by the issuers or the guarantee agency. As the issuers need to redistribute the cash inflow from a mortgage pool to a series of bond classes with different principals and maturities, the spirit of their risk management should focus on the mismatch between the cash flows of mortgages and the MBS. Moreover, the possibility of losing money from the guarantee of principal and interest payments in the selected MBS pools is a concern.

As in all other types of business, issuers face risks on the first day placing MBS in the market. MBS issuers face three basic types of risk, namely reinvestment risk, cash flow mismatch between the mortgage pool and the MBS, and the potential loss of subordinate MBS. When the prepayment rate is high, the excess cash that is received from the mortgage pool suffers a lower return rate than does the coupon rate of mortgages. After the prepayment principal is received from the underlying mortgage pool, the issuers may receive more money than expected, and thus create the reinvestment risk. The cash flow mismatch between the mortgage pool and the MBS will be occurred if the issuers cannot predict the embedded options of mortgage loans accurately. For potential loss of subordinate MBS, the issuers will absorb the cash flow shortfalls that are caused by

default-related losses with respect to the senior tranches when the quality of the mortgages is poor.

Many existing models of mortgages and MBS adopt the arbitrage principle of option pricing theory (i.e. they consider a mortgage as a bond with prepayment, delinquency, and default options). Keenan, Muller, and Epperson (1993) adopted the traditional approach, but modified it with a decision tree to define prepayment and delinquency probability. Ambrose, Buttimer, and Capone (1997) explored the time delay between delinquency and default, and the chance that the mortgage might recover under the delinquency state, both of which are not considered by most studies.

My model is a blend of those from previous studies, with extensions. The objective of this paper is to develop a risk management tool for issuers and guarantors to monitor their MBS portfolios. I adopt a similar approach to previous studies, but use a simulation program to generate the random paths on the uncertainty. I will generate different paths of cash flow scenarios through computer programs, and for each cash flow will generate the whole cash flows with pre-defined variables.

Another feature of the model is that it combines the prepayment, delinquency, default, and recovery of delinquency into a single model. Many previous mortgage and MBS models treated prepayment and default in discrete models. Yet, according to the discussion of Deng, Quigley, and Van Order (2000), the simultaneity of these options is important in explaining mortgage behavior. Factors that trigger an option are important in triggering or preceding the exercise of another option, and we should treat all the options together. Without amalgamating the effect of all options, we cannot precisely predict the mortgage behavior to furnish a complete and comprehensive model.

The traditional method of mortgage pricing calculation projects the simple cash flow, which may not be accurate enough to determine the MBS pricing. The model that is developed in this paper projects the cash flow for both sides of MBS (i.e. the cash inflow from the mortgage pool and the cash outflow to the MBS portfolio). The model will identify the potential loss of guarantee payment and the reinvestment risk on excess cash flow. The model is based on the structuring model of the Korea Mortgage Corporation (KoMoCo), and seven MBS from 2000-1 to 2001-2 are tested. By exploiting the development of the structuring model for KoMoCo, the model is appropriated for issuers to manage the risk elements of their MBS portfolios.

FACTORS IN PROJECTING THE MORTGAGE CASH FLOW

As stated in the previous section, whether MBS issuers can accurately project the cash flow of mortgage portfolios is particularly important in terms of risk management. The following are the key factors in projecting the cash flow of mortgages.

Prepayment

Prepayment is a critical factor for evaluating the performance and risk management of MBS. When the mortgage is prepaid, the prepayments of principal significantly alter the cash flow that is received from the principal and interest of the mortgage pool. If mortgages are prepaid when the interest rate trend is going down, then the funds that are received from prepaid loans will attain a lower reinvestment rate than their original mortgage contract rate. Therefore, investors and issuers suffer reinvestment losses by forgoing the opportunity to earn the high interest income from the original contract.

Delinquency and Default

Delinquency and default also affect the cash flow of the mortgage portfolio. Unlike prepayment, delinquency and default may take several months to complete. Therefore, we need to consider the whole process flow in cash flow management, such as the time gap between delinquency and liquidation, the cure speed table, and the loss severity that is equal to the percentage of loss on the defaulted loan balance by the lender. When the liquidation collection minus transaction costs is less than the outstanding balance of the mortgage, the lender will suffer a loss that is equal to the multiple of the defaulted balance to the loss severity.

Recovery of delinquency and cure speed

In real life situations, only a small percentage of delinquent loans default, while the majority are recovered to the normal state. One of the common approaches to describing the recovery of delinquency for mortgages is the determination of the “cure speed” (i.e. the rate of recovery of a delinquent mortgage to the normal state). With the figures of the cure speed table, one can determine the ratio of the pool to proceed to $(i+1)$ months delinquency and recover to the normal state in the status of i month delinquency.

THE METHODOLOGY

Various previous studies explored the embedded options of mortgages, but most of them explored the option separately and did not combine the prepayment, delinquency, default, and recovery of delinquency into a single model. My model attempts to combine all of these factors, because they are interrelated. For example, when the prepayment option is exercised, the default option becomes ineffective. Similarly, when the delinquency option is exercised, the probability of exercising the default option will increase with the length of delinquency.

The primary scenario begins with the simplest cases of mortgage payment without embedded options (i.e. all mortgages follow the normal schedule installment until maturity). A distinct schedule can be calculated through a simple formula that includes the original balance and the interest rate and maturity of each individual loan, and then can be aggregated to form the cash flow of the whole mortgage pool.

The subsequent task is to incorporate the embedded options into the analysis. When the embedded option is exercised, the cash flow of the mortgage is affected, and has a shorter maturity than originally expected. The model links the property price $H(i)$, interest rate $r(i)$, prepayment probability $\pi(r, H, i)$, delinquency function $\lambda(r, H, i)$, and default function $\delta(r, H, i)$ to build the picture of the scheduled cash flow. I will develop a Monte Carlo simulation program to generate the cash flow of a single mortgage, and use it to determine the cash flow of all the mortgages

Finally, the projection of cash inflow from mortgages will be combined with the cash outflow of the MBS, as shown in Fig. 2. The model includes other factors of the cash flow such as the call option of issuers and the reinvestment schedule of excess cash.

Figure 1: Cash Flow Projection of Mortgages

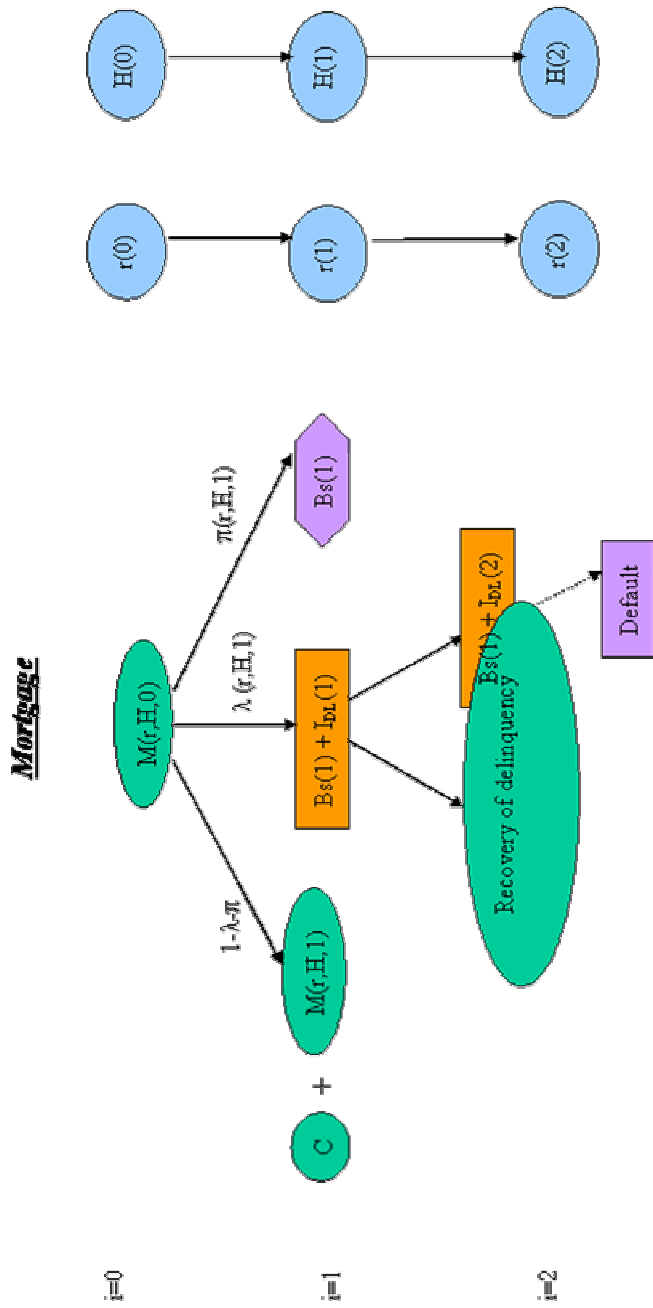
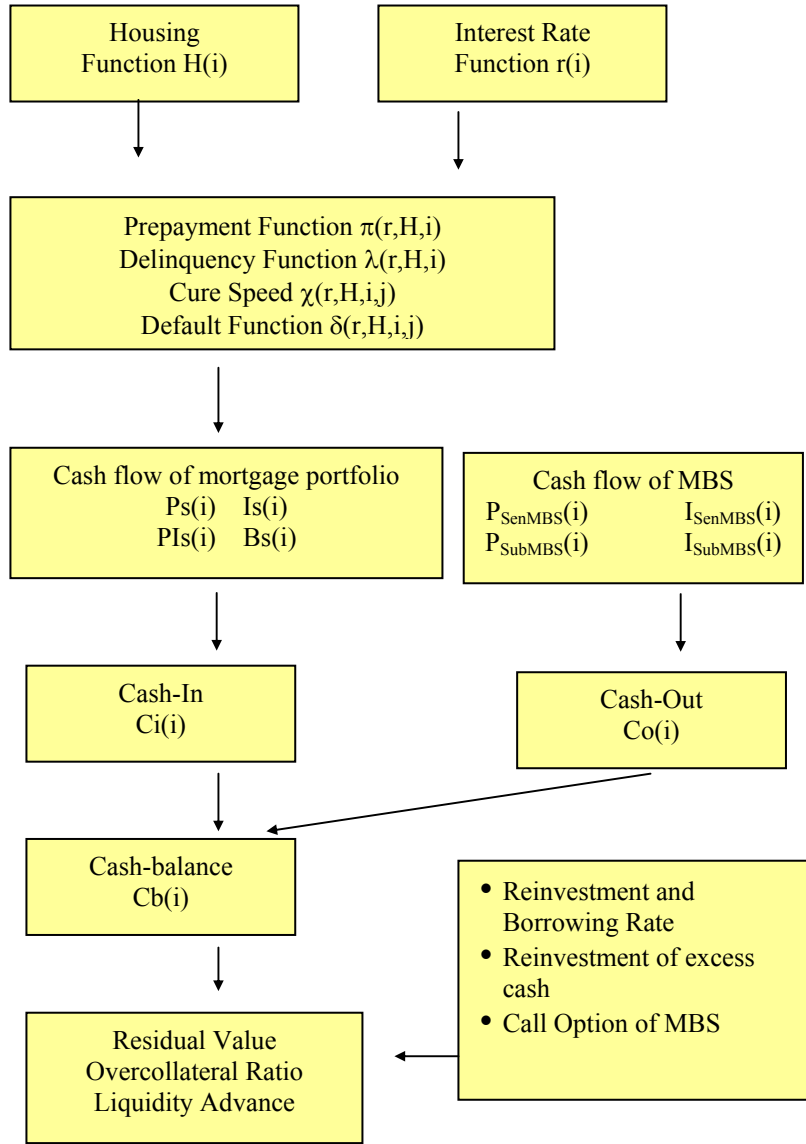


Figure 2: Combined Structure of Mortgages and the MBS



Interest
Rate

Underly
Asset

THE MODEL

The following notation will be used in the model:

i = the number of installment payments
 $t(i)$ = the date of the i th monthly payment

Interest Rate

$r(i)$ = the interest rate function
 $im(i)$ = the contract rate of mortgage
 $id(i)$ = the delinquency interest rate
 $ir(i)$ = the reinvestment rate from period i to $i+1$
 $ib(i)$ = the borrowing rate from period i to $i+1$

Conditional Probability of Prepayment, Delinquency, and Default

(r,H,i) = the conditional probability of prepayment
 (r,H,i) = the conditional probability of delinquency
 (r,H,i,j) = the Cure Speed rate at the j th payment which occurs at the i th payment
 (r,H,i,j) = the conditional probability of default at the j th payment which occurs at the i th payment

Original Cash Flow (the cash flow of the mortgage without considering prepayment, delinquency, and default)

$Po(i)$ = the original schedule of principal payment
 $Io(i)$ = the original schedule of interest payment
 $PIo(i)$ = the original schedule of principal + interest payment
 $Bo(i)$ = the original schedule of outstanding balance

Scheduled Cash Flow (the cash flow of mortgage with prepayment, delinquency, and default)

$Ps(i)$ = the schedule of principal payment with prepayment, delinquency, and default
 $Is(i)$ = the schedule of interest payment with prepayment, delinquency, and default
 $PIs(i)$ = the schedule of principal + interest payment with prepayment, delinquency, and default
 $Bs(i)$ = the schedule of outstanding balance with prepayment, delinquency, and default

$PPREP(i)$ = the prepayment Principal (both full and partial)
 $PDLC(i)$ = the Delinquency Principal in current period
 $PDLT(i)$ = the Accumulated Delinquency Principal

PDLR(i) = the recovery of delinquency principal according to the
Cure Speed
IDL(i) = the Delinquency Interest
PDF(i) = the Default Principal

Combined Cash Flow of the Mortgage and the MBS
Ci(i) = the cash inflow from the mortgage pool in period i
Co(i) = the cash outflow to the MBS in period i (the timely
payment of principal and interest)
Cb(i) = the cash balance in period i

Cash Outflow to MBS
PSenMBS(i) = the principal payment to Senior MBS
ISenMBS(i) = the interest payment to Senior MBS
PSubMBS(i) = the principal payment to Subordinate MBS
ISubMBS(i) = the interest payment to Subordinate MBS

Other Variables
T = the tenor of the mortgage
H(i) = the housing price of the underlying asset

Underlying Property Price and Interest Rate

The model follows the assumption of asset price movement as a lognormal random walk Brownian motion in the option-pricing model of Black and Scholes (1973) and Merton (1973). The asset price is expressed as a lognormal random walk with constant volatility, which is a specific category of the Markovian process. As suggested by Kau (1995), the value of the underlying property is assumed to follow the Brownish motion (i.e. the change in the process over the next time interval is independent of what happened in the past):

$$\frac{dH}{H} = (u - b)dt + \sigma_H dz_H \quad (1)$$

where $u - b$ = the expected rate of housing return
 H^2 = the instantaneous variance of housing return
 dz_H = the Standard Wiener Process

Interest rate forecasting is important, as interest rates affect the pricing of MBS. Some models assume that the interest rate function is a log-normal random walk with a drift that centers the distribution on the implied forward

rate. Similar to the previous assumption on property price, I adopted the mean-reverting normal process to define the interest rate r , and the model adopts the mean-reverting process to describe the interest rate function:

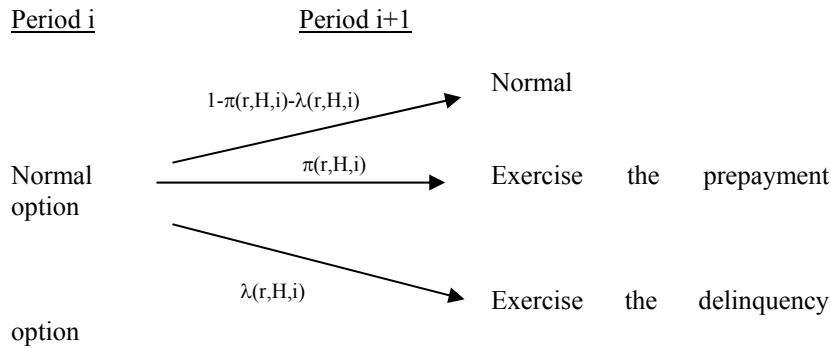
$$dr = \gamma(\theta - r)dt + \delta r \sqrt{r} dz_r \quad (2)$$

- where
- γ = the speed of adjustment
 - θ = the long-term value for the interest rate
 - σ_H^2 = the instantaneous variance of the interest rate
 - dz_H = the Standard Wiener Process

The reinvestment rate $ir(i)$ and borrowing rate $ib(i)$ can be determined through this mean-reverting process. If the mortgage is ARM, then the contract interest rate of mortgage $r(i)$ can also be determined through this method.

Decision Tree of a mortgage loan

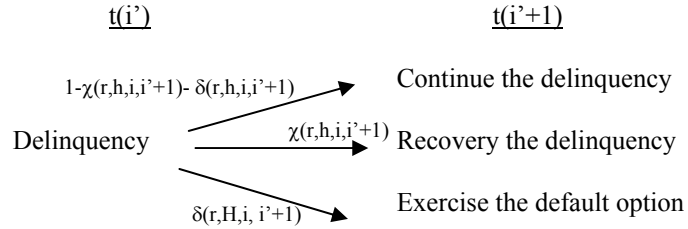
With the embedded options of prepayment, delinquency, and default, we can consider the mortgage as a decision tree with different branches of decision marking when a particular option is exercised. For the normal state of period i , the borrower can either exercise the prepayment or delinquency options, as in the following diagram:



For each transition period from i to $i+1$, we assume that the mortgage will have the probability distribution of:

- | | |
|-----------------------------|-----------------------------------|
| Normally scheduled payment: | $1 - \pi(r,H,i) - \lambda(r,H,i)$ |
| Prepayment: | $\pi(r,H,i)$ |
| Delinquency: | $\lambda(r,H,i)$ |

When the prepayment option is exercised, the cash flow of the mortgage is terminated and the lender receives the entire outstanding principal of the loan, plus penalty charges and interest if there are any available. Alternatively, when the delinquency option is exercised, the mortgage either recovers its normal status according to the cure rate, continues the delinquency, or turns to default as shown in the following decision tree (assuming that the delinquency occurs in period i):



Continue the delinquency: $1 - \chi(r,h,i,i'+1) - \delta(r,h,i,i'+1)$
 Recovery with cure speed: $\chi(r,h,i,i'+1)$
 Default: $\delta(r,H,i,i'+1)$

When the delinquency continues from period i' to $i'+1$, the delinquency principal is accumulated with additional delinquency interest. Of course, the borrower can also recover the delinquency by paying all of the unpaid delinquency principal to the lender, or by exercising the default option to terminate the cash flow of the mortgage.

Schedule of Mortgage Cash Flow without Embedded Options

If the embedded option of prepayment, delinquency, and default is not considered, then the amortized schedule and cash flow of the mortgage can be determined by the following formula:

$$PI_o(i) = Bo(i-1) \frac{(i_r(i)/12)(1+i_r(i)/12)^{N-i+1}}{(1+i_r(i)/12)^{N-i+1} - 1} \tag{3}$$

$Io(i) = Bo(i-1) \times ir(i) \times (t(i) - t(i-1))$
 $Po(i) = PIo(i) - Io(i)$
 $Bo(i) = Bo(i-1) - Po(i)$

Schedule of Mortgage Cash Flow with Embedded Options

When the embedded options of prepayment, delinquency, and default are considered, the revised cash flow schedule will be determined as follows:

(i) Prepayment:

$$PPREP(i) = Bs(i-1) \times \pi(r, H, i)$$

(ii) Delinquency:

$$PDLC(i) = Ps(i-1) \times \lambda(r, H, i)$$

$$PDLT(i) = PDLT(i-1) + PDLC(i) - PDLR(i)$$

$$IDL(i) = PDLT(i-1) \times id(i) \times (t(i) - t(i-1))$$

$$P_{DLR}(i) = \sum_{\alpha=1}^{i-1} P_{DLC}(\alpha) \lambda(r, H, \alpha, i - i) \quad (4)$$

$$P_{DF}(i) = \sum_{\alpha=1}^{i-1} \sum_{\beta=\alpha}^{i-1} P_{DLC}(\alpha) \lambda(r, H, \alpha, \beta) \delta(r, H, \beta, i) \quad (5)$$

(iii) Default:

(iv) Scheduled Cash Flow with Embedded Options:

$$Ps(i) = Bs(i-1) \times [Po(i)/Bo(i-1) + \pi(r, H, i)] - PDLC(i) + PDLR(i)$$

$$Is(i) = [Bs(i-1) - PDLC(i) - PDF(i)] \times [Io(i)/Bo(i-1)] + IDL(i)$$

$$Pls(i) = Ps(i) + Is(i)$$

$$Bs(i) = Bs(i-1) - Ps(i) - PDF(i)$$

Combined Cash Flow with MBS

It is the duty of issuers to provide guarantees of the timely payment of principal and interest to the senior portion of the MBS. As issuers suffer losses only when the funds that are received from the mortgage pool (P&I) cannot cover the payment of the senior MBS, the guarantee fee should equal the expected value of insufficient fund coverage. According to the common practice of the MBS industry, only the principal and interest payment of senior MBS are guaranteed by issuers. Investors in subordinate MBS receive the principal and interest payment only when the payment of senior MBS is fully settled (i.e. when the cash inflow from the mortgage portfolio can satisfy all of the principal and interest payments of the senior MBS, when the Collection Account Balance at period i-1, including reinvestment or borrowing interest, plus when the Cash inflow in period i is greater than the principal and interest payment of the senior MBS). The payment of the subordinate MBS is illustrated by the following formula:

If $Cb(i-1) \geq 0$ (i.e. a positive collection account balance at period i-1):

Case 1: $C_b(i-1) (1+ir(i-1)) + C_i(i) - (P_{SenMBS}(i) + I_{SenMBS}(i)) < 0$

where $ir(i-1)$ = the reinvestment rate from period $i-1$ to i

$\Rightarrow P_{SubMBS}(i) = I_{SubMBS}(i) = 0$

$Co(i) = P_{SenMBS}(i) + I_{SenMBS}(i)$

$C_b(i) = C_b(i-1) (1+ir(i-1)) + C_i(i) - P_{SenMBS}(i) + I_{SenMBS}(i)$

No payment on the principal and interest of the subordinate MBS. The issuer needs to borrow money to pay the principal and interest of the senior MBS.

Case 2: $0 \leq C_b(i-1)(1+ir(i-1))+C_i(i)- (P_{SenMBS}(i)+I_{SenMBS}(i)) < P_{SubMBS}(i) + I_{SubMBS}(i)$

$\Rightarrow P_{SubMBS}(i) + I_{SubMBS}(i) = C_b(i-1)(1+ir(i-1))+C_i(i)- P_{SenMBS}(i)+I_{SenMBS}(i)$

$Co(i) = C_b(i-1) (1+ir(i-1)) + C_i(i)$

$C_b(i) = 0$

Only a partial amount of principal and interest will be paid to investors in the subordinate MBS.

Case 3: $C_b(i-1) (1+ir(i-1)) + C_i(i) - (P_{SenMBS}(i) + I_{SenMBS}(i)) \geq P_{SubMBS}(i) + I_{SubMBS}(i)$

$\Rightarrow Co(i) = P_{SenMBS}(i) + I_{SenMBS}(i) + P_{SubMBS}(i) + I_{SubMBS}(i)$

$C_b(i) = C_b(i-1) (1+ir(i-1)) + C_i(i) - Co(i)$

The full amount of principal and interest will be paid to investors in the senior and subordinate MBS.

If $C_b(i-1) < 0$ (i.e. a negative collection account balance at period $i-1$):

Case 1: $C_b(i-1) (1+ib(i-1)) + C_i(i) - (P_{SenMBS}(i) + I_{SenMBS}(i)) < 0$

where $ib(i-1)$ = the borrowing rate from period $i-1$ to i

$\Rightarrow P_{SubMBS}(i) = I_{SubMBS}(i) = 0$

$Co(i) = P_{SenMBS}(i) + I_{SenMBS}(i)$

$C_b(i) = C_b(i-1) (1+ib(i-1)) + C_i(i) - P_{SenMBS}(i) + I_{SenMBS}(i)$

No payment on the principal and interest of the subordinate MBS. The issuer needs to borrow money to pay the principal and interest of the senior MBS.

Case 2: $0 \leq C_b(i-1)(1+ib(i-1))+C_i(i)- (P_{SenMBS}(i)+I_{SenMBS}(i)) < P_{SubMBS}(i) + I_{SubMBS}(i)$

$\Rightarrow P_{SubMBS}(i) + I_{SubMBS}(i) = C_b(i-1)(1+ir(i-1))+C_i(i)- P_{SenMBS}(i)+I_{SenMBS}(i)$

$Co(i) = C_b(i-1) (1+ib(i-1)) + C_i(i)$

$C_b(i) = 0$

Only a partial amount of principal and interest will be paid to investors in the subordinate MBS.

$$\begin{aligned} \text{Case 3: } & Cb(i-1)(1+ib(i-1)) + Ci(i) - (PSenMBS(i) + ISenMBS(i)) \geq \\ & PSubMBS(i) + ISubMBS(i) \\ \Rightarrow & Co(i) = PSenMBS(i) + ISenMBS(i) + PSubMBS(i) + ISubMBS(i) \\ & Cb(i) = Cb(i-1)(1+ib(i-1)) + Ci(i) - Co(i) \end{aligned}$$

The full amount of principal and interest will be paid to investors in the senior and subordinate MBS.

According to the calculation in the above model, MBS issuers can consider the following important factors when they determine the risk levels of their MBS portfolios.

Exercise of MBS Call Options

Issuer's call options are common in the MBS market. There are two types of call option for MBS issuers:

- (i) a pre-mature call option – the issuer has the right to redeem the outstanding bonds under a pre-defined period and condition; and
- (ii) an issuer's clean-up call – the issuer can fully redeem the outstanding notes on any monthly MBS Payment Date, on which the outstanding mortgage pool balance is a certain percentage less than the initial mortgage pool amount. In market practice, the issuer's clean-up call is usually set at 10% of the initial mortgage pool amount.

Call options are among the influential tools that issuers use to manage their MBS portfolios. When there is a financial benefit to redeem the MBS, issuers will exercise a call option to enhance the return of the total portfolio. The call option of KoMoCo papers is the pre-mature option, in which the issuer can redeem the paper prior to the maturity date. The senior MBS are callable in the sequence of any coupon payment date, providing that those with shorter maturities are fully repaid, and the exercise date is later or equal to the legal maturity date of the previous tranche. The installment tranche is callable independently or collectively at any quarterly coupon payment date that falls on or after five years. When the call option is exercised, the available cash is distributed to the National Housing Fund at any coupon payment date once the Senior MBS are fully prepaid.

The judgment of call option exercise is challenging. When MBS issuers need to decide whether to exercise a call option, they compare the financial benefits of both situations with sophisticated forecasting models that generate the cash flow projections of both scenarios. I have developed a program with which issuers can compare the difference and determine the best decision to enhance their portfolio returns.

Figure 3: Variables for MBS Risk Measurement

Factors	Variables	Measurement
Collection Account Balance	$C_b(i)$	Indicates the outstanding cash amount and the liquidity risk of the portfolio.
Overcollateralization Ratio	$(C_b(i) - C_o(i) - I_{SenMBS(i)}) / P_{SenMBS(i)}$	Indicates whether the portfolio is in excess or in shortage of cash inflow from the mortgage pool.
Net Residual Value	Value of $C_b(N)$ at the maturity of the MBS	If the residual is held by the issuer, it will suffer a loss if the Net Residual Value is negative at the maturity date of the MBS.
Liquidity Advance	The amount of borrowing when $C_b(i)$	The issuer needs to borrow money from other sources to pay the principal and interest of the MBS if the liquidity advance is positive.

EMPIRICAL ANALYSIS

The Data

The data for this model was provided by KoMoCo with seven MBS portfolios that were issued from 2000-1 to 2002-2. The data set contains 350,139 mortgages that were held by the National Housing Fund (NHF), except for MBS 2002-1, which is held by Samsung. For each mortgage, I obtained the data of the original outstanding balance, the coupon rate, and the maturity date, so that the scheduled balance of the portfolio can be calculated. The file was downloaded from the computer system of KoMoCo, and extracted from a personal computer file for analysis.

I also obtained the 3-month CD rate for the past ten years for projecting the reinvestment and borrowing rates of the model. The historical data of prepayment, delinquency, default, and cure speed was also available for analyzing the function of these embedded options. I used this data to project the cash flow of the mortgage portfolio, and combined it with the cash flow of the MBS pool to determine the risk level of the company.

Figure 4: MBS Issued by the Korea Mortgage Corporation

Issuer	Issue Date	Total Issue Amount (bn Won)	Senior MBS (Public Offering) (billion Won)	Sub MBS (Private Placement) (billion Won)		No. of Mortgages	
		bn (won)	bn (won)	No of Tranches (Maturity)	bn (won)	No of Tranches (Maturity)	
MBS 2000-1	7/4/2000	397.6	369	9 (6mths~6yr)	28.6	2 (6yr, 7yr)	82,570
MBS 2000-2	1/9/2000	500	479	11 (6mths~7½yr)	21	2 (8yr, 9yr)	103,819
MBS 2000-3	8/12/2000	381.3	368	12 (6mths~10yr)	13.3	1 (11.5yr)	58,367
MBS 2001-1	18/5/2001	237.7	228	12 (6mths~10yr)	9.7	1 (12yr)	15,633
MBS 2001-2	20/9/2001	505	500	7 (1yr~14yr)	5	1 (15yr)	44,750
MBS 2002-1	23/1/2002	17.97	17.5	3 (1yr~7.5yr)	0.47 billion	1 (15yr)	601
MBS 2002-2	21/2/2002	510.2	492	8 (0.5yr~14yr)	18.2	1 (15yr)	44,399
Total*		2549.77 (USD 1.98 bn)	2,453.50 (USD 1.90 billion)		96.27 billion won (USD 74.62 million)		350,139

*Assume 1 USD = 1290 Korean Won

Simulation Results

A comprehensive program has been developed for handling the calculation of the mortgage cash flow, the MBS principal and interest payment, the balance of the collection account, and the simulation of multiple path analysis. The effects of prepayment, delinquency, default, the reinvestment rate, and the loss severity are analyzed with the risk elements of issuers. The risk factors of Collection Account Balance, Overcollateralization Ratio, Net Residual Value, and Liquidity Advance are determined by the program under different scenarios.

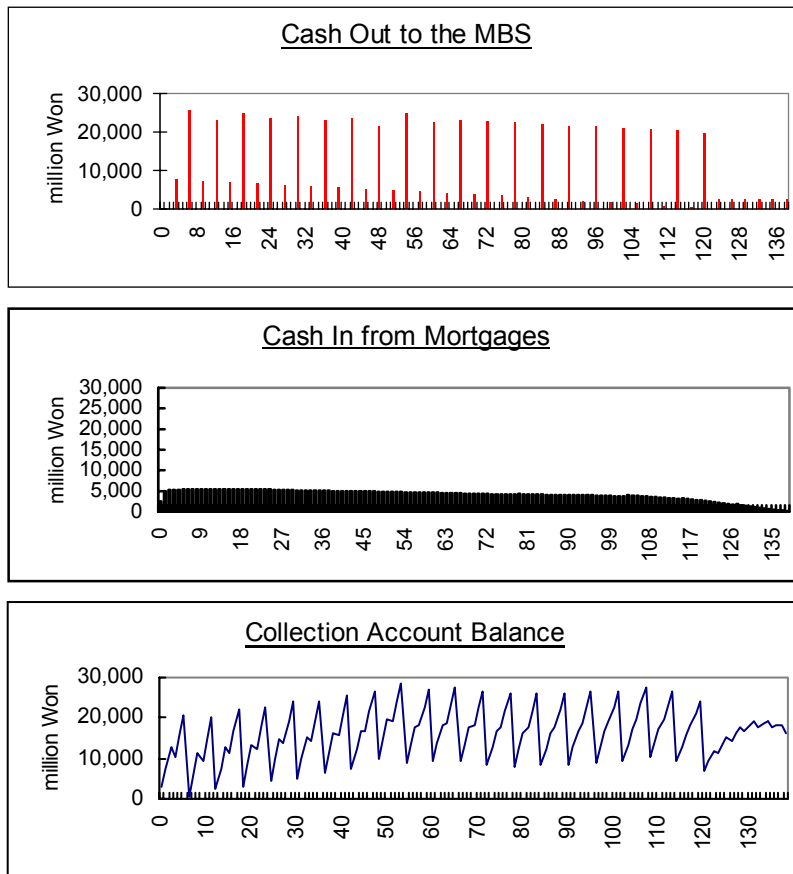
In order to weigh up the performance of the program, I imported the historical data of KoMoCo to the program, and the result was satisfactory. The output of the analysis is highly recognized by KoMoCo. It allows the staff to project the cash flow of their portfolio, and helps them to explore the embedded risk of their MBS issues. For some MBS tranches with earlier issue dates, such as MBS 2000-1 to MBS 2000-3, the simulation

result is compared with the actual figures of the years 2001 and 2002, and the outcome has confirmed the accuracy of the program. Since the timeframe of the actual figures is only for two years, this comparison may not be statistically significant enough to draw any conclusion, and will not be discussed in this paper.

Collection Account Balance

Fig. 5 reports the results of projecting the cash flow of the mortgage portfolio, the MBS portfolio, and the balance of the collection account on the selected portfolio of KoMoCo. The cash inflow from mortgages is received on a monthly basis, and the cash outflow to the MBS for coupon interest is quarterly, and the principal is biannual.

Figure 5: The Simulation Result for the Collection Account Balance

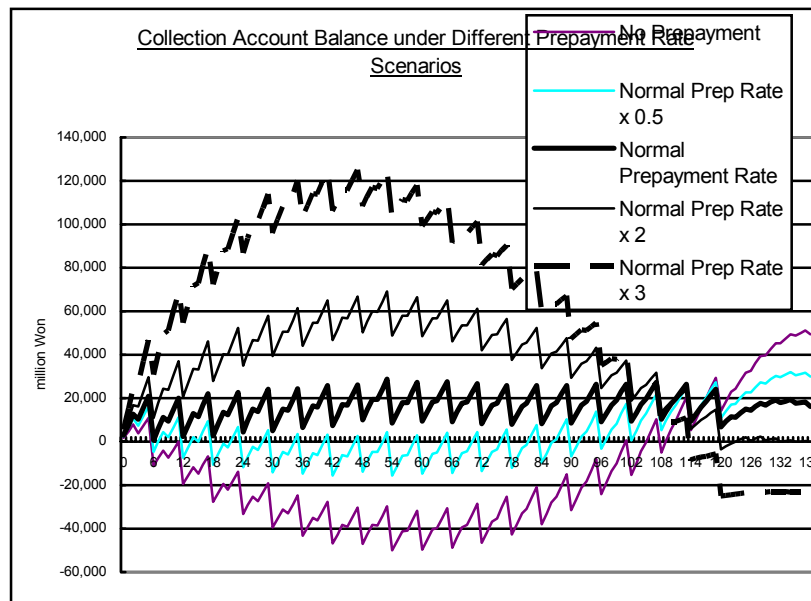


Due to the mismatch between the payment sequence of the mortgages and the MBS portfolio, the balance of the collection account is in a “jigsaw” shape. This diagram is important to the MBS issuers as it can show:

- (i) the variance between the cash flow of the mortgages and the MBS portfolio;
- (ii) the possibility of liquidity advance (i.e. the balance of the collection account being less than zero);
- (iii) whether the cash inflow from mortgages is enough to pay the cash outflow to the MBS; and
- (iv) the Net Residual Value (i.e. the value of the collection account balance at maturity).

Fig. 6 demonstrates the effect of prepayment on the collection account balance. The prepayment rate is determined according to the analysis of historical data from the mortgage portfolio of KoMoCo. The result shows that the “jigsaw” shape of the collection account balance will bow upward with high prepayment and downward with low prepayment.

Figure 6: The Collection Account Balance under Different Prepayment Rate Scenarios



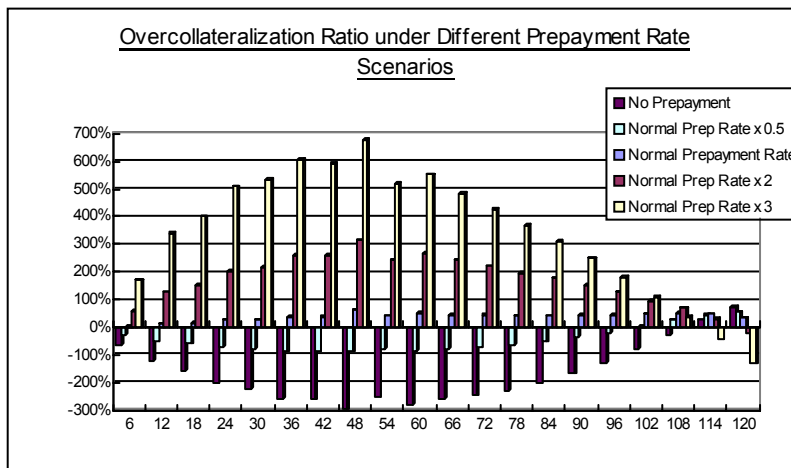
When the prepayment rate is higher than expected, the issuers will receive more cash from the mortgage portfolio in the earlier stage so that it will create a higher collection account balance in the middle of the period, and a

lesser balance at the later stage. Given that the return on reinvestment is usually lower than the mortgage coupon rate, a high prepayment will generate a lower or even negative Net Residual Value.

The Overcollateralization Ratio

Fig. 7 presents the relationship between the prepayment rate and the overcollateralization ratio. Similar to the situation of the collection account balance, the overcollateralization ratio will bow upward with high prepayment and downward with low prepayment. When the value of the overcollateralization ratio is negative, the balance of the collection account will not be sufficient to pay the principal and interest of the MBS.

Figure 7: The Overcollateralization Ratio under Different Prepayment Rate Scenarios



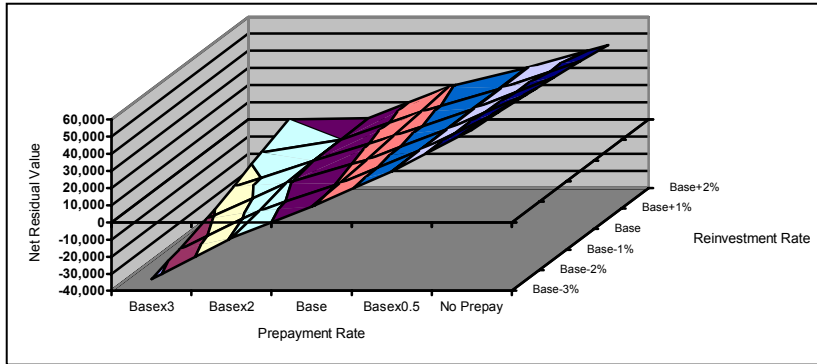
Net Residual Value

Fig. 8 presents the relationship among the Net Residual Value, the prepayment rate, and the reinvestment rate. The prepayment rate is determined according to the analysis of historical data from the mortgage portfolio of KoMoCo, and the reinvestment rate is generated by a simulation program through the mean-reverting normal process. The result shows that the Net Residual Value increases with the reinvestment rate and decreases with the prepayment rate.

In real life situations, the prepayment rate is negatively correlated with the reinvestment rate (i.e. the prepayment increases when the reinvestment decreases). When the interest rate falls, borrowers have a good opportunity

to obtain lower mortgage rates than the original contract rate. Thus, if the interest rate trend is going down, then the reinvestment rate will decrease, the prepayment rate will increase, and the Net Residual Value will significantly diminish.

Figure 8: The Net Residual Value against the Prepayment and Reinvestment Rates

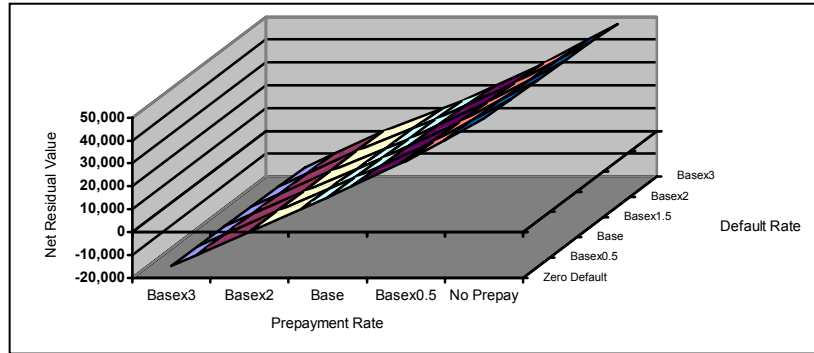


Reinvestment Rate/ Prepayment Rate	Base -3%	Base -2%	Base -1%	Base	Base + 1%	Base + 2%
Base Prep Rate x 3	-33,158	-28,101	-22,173	-15,263	-7,250	0
Base Prep Rate x 2	-9,021	-5,374	-1,152	0	0	867
Base Prepayment Rate	9,172	10,716	12,451	14,397	16,581	19,031
Base Prep Rate x 0.5	29,720	29,935	30,106	30,229	30,296	30,301
No Prepayment	53,892	52,484	50,765	48,697	46,240	43,348

(unit: million Korean Won)

Fig. 9 further presents the relationship among the Net Residual Value, the prepayment rate, and the default rate. When the default rate increases, the Net Residual Value decreases. As the default rate is usually maintained at a low level in Korea, the effect of default on the Net Residual Value is not as significant as the effect it has on prepayment.

Figure 9: The Net Residual Value against the Prepayment and Default Rates



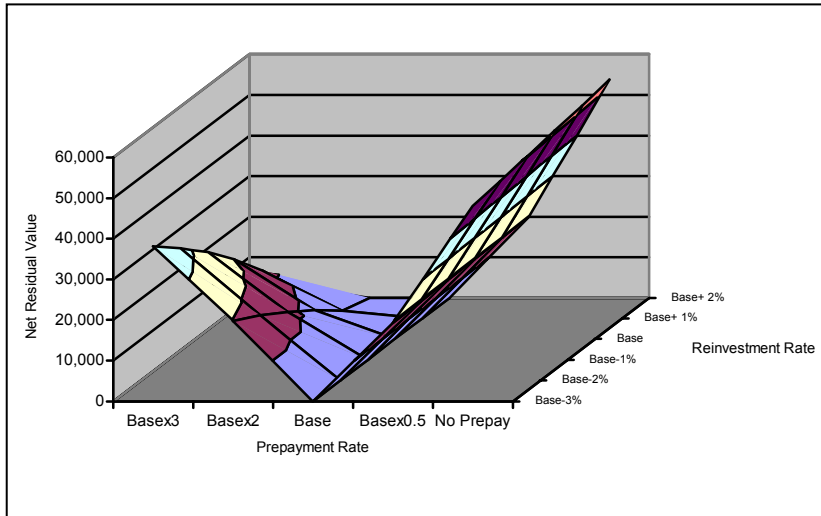
Default Rate/ Prepayment Rate	Zero Default	Base x 0.5	Base	Base x 1.5	Base x 2	Base x 3
Base Prep Rate x 3	-14,917	-15,090	-15,263	-15,436	-15,609	-15,955
Base Prep Rate x 2	0	0	0	0	0	0
Base Prepayment Rate	15,048	14,723	14,397	14,072	13,746	13,095
Base Prep Rate x 0.5	31,007	30,618	30,229	29,839	29,450	28,671
No Prepayment	49,637	49,167	48,697	48,227	47,757	46,816

(unit: million Korean Won)

Liquidity Advance

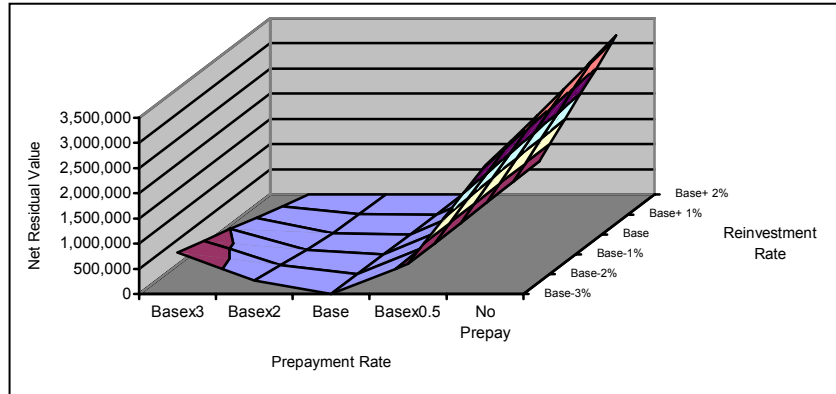
Figs. 10 and 11 report the relationship among the Liquidity Advance, the prepayment rate, and the reinvestment rate. Fig. 10 shows the maximum value of the Liquidity Advance, and Fig. 11 shows the aggregate value. The results show that the Liquidity Advance and prepayment rate have a convex curve relationship (i.e. the curve has a minimum at the base situation, and either the increase or decrease in prepayment rate will have a higher liquidity advance). When the prepayment rate increases, the issuers will receive more cash at the earlier stage, but a lesser return from reinvestment and thus a high liquidity advance at the later stage. When the prepayment rate decreases, the collection account will have an insufficient balance to pay the MBS principal at the earlier stage, and this will result in a liquidity advance.

Figure 10: The Maximum Liquidity Advance against the Prepayment and Default Rates



(million Korean Won)	Base -3%	Base -2%	Base -1%	Base	Base + 1%	Base + 2%
Base Prep Rate x 3	38,159	32,674	26,500	19,568	11,800	3,112
Base Prep Rate x 2	19,815	16,042	11,823	7,112	1,861	0
Base Prepayment Rate	0	0	0	0	0	0
Base Prep Rate x 0.5	19,233	19,422	19,614	19,808	20,005	20,204
No Prepayment	48,097	49,136	50,206	51,308	52,442	53,714

Figure 11: The Total Liquidity Advance against the Prepayment and Default Rates



(million Korean Won)	Base - 3%	Base - 2%	Base - 1%	Base	Base + 1%	Base + 2%
Base Prep Rate x 3	828,751	663,303	500,287	327,542	161,719	9,885
Base Prep Rate x 2	270,634	179,910	88,836	26,472	2,427	0
Base Prepayment Rate	0	0	0	0	0	0
Base Prep Rate x 0.5	596,736	609,482	623,112	637,602	652,815	668,768
No Prepayment	4	7	4	1	0	2

Conclusions

This paper has presented a unified model of risk management for MBS issuers. By adopting the structuring model of KoMoCo, I have projected the cash inflow from a mortgage pool and the cash outflow of an MBS. The model alters previous efforts through the addition of decision trees and the embedded options of prepayment, delinquency, and default in a single and consolidated model. Moreover, the time delay between delinquency and default and the probability of recovery under the delinquency state have also been considered. Combining the results of the model with the actual data of KoMoCo, I have established a prudent framework for MBS issuers who wish to assess the risk management of their MBS portfolios.

This paper illustrates several important factors in the management of MBS portfolios. MBS issuers should always:

Sustain a diminutive mismatch between the cash inflow and outflow to retain a low balance in the collection account. If the collection account balance is too high, then it will introduce a high reinvestment risk to the issuer. Conversely, if the collection account balance is too low, then it may bring about the probability of a liquidity advance.

Maintain their portfolios at an appropriate overcollateralization ratio to prevent an immense surplus or shortfall for the payment of the MBS principal.

Always prevent a negative Net Residual Value.

Consider exercising their call options to prevent the profusion of the collection account balance when the prepayment rate is more than the original expectation.

Although the results will be useful for the risk management of MBS issuers, there are some limitations in the model, making further research necessary. The financial factor is only one of many aspects in the risk management of MBS. Risk management is a widespread subject that involves operational risk and interest risk, etc., and we cannot only consider financial factors in the analysis. When MBS issuers make a important decision, such as whether to exercise a call option, the decision involves the strategic position of the corporation rather than just the financial benefit. Therefore, we should consider more factors when managing the risk of MBS portfolios.

The model itself is flexible and can also be used to facilitate the risk pricing of MBS guarantors and investors. Indeed, more types of MBS will be considered with the model in future studies..

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