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Determinant of Allocation of Housing Inventory: Competition between Households and Investors

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This paper develops a theoretical model for equilibrium rent-to-price ratios from the competition between households and investors in the housing market. Households make their housing tenure choice in terms of rent vs. buy such as minimizing the cost of occupying a housing unit. On the other hand, investors choose between investing in rental housing vs. other investment opportunities in order to maximize their net present value. In the face of limited housing inventory, households and investors bid against one another which determines the allocation of the housing units among households (owner occupied properties) and investors (rental properties). We derive the sensitivity of the equilibrium rent-to-price ratio with respect to various market parameters, and subsequently analyze their potential impacts on the homeownership rate in the community. We show that some government mortgage programs subsidize homeownership to increase the affordability of owning a house, but may also provide even more incentive to the housing investors. Unless the government can effectively control the eligibility of borrowers, such affordable mortgage programs could work against their objectives and lead to higher housing prices and lower homeownership rates. Our model framework can be used to analyze the potential impacts of some of the new affordable housing policies on house prices or homeownership rates before adopting them.

Keywords

Housing Tenure Choice, Rental Property, Rent-to-price Ratio, Reservation Price, Homeownership Rate, Affordable Housing Policy.

1.   Introduction

Housing tenure choice and homeownership rates have received considerable attention from real estate researchers, yet they have been generally analyzed as two separate issues. A microeconomic model of individual tenure choice has not been linked to macroeconomic homeownership rate yet. In this paper, we provide an introduction to this linkage by starting with the tenure-choice decisions of individual households and, through their aggregation, develop an equilibrium rent-to-price ratio (R/P ratio) for clearing the rental housing market. We thus back into the community homeownership rate by specifying demand for and supply of rental housing in terms of R/P ratios as the price equilibrating mechanism in the market. In the process, households self-select between renting or owning based on their unique R/P ratio of reservation prices. Since homebuyers and investors are competing for the same finite housing stock, our theoretical model yields the overall homeownership rate for the community.

A mathematical model is developed for an economy that is composed of homogeneous investors and a set of heterogeneous households that differ only by their expected lengths of housing tenure. As shown by Capone (1995), the expected housing tenure length is sufficient for separating otherwise homogeneous households into owners and renters. The model is flexible enough to facilitate the observation of economies with and without mortgage credit facilities for owners and investors. This also shows the impacts on homeownership rates due to changes in economic conditions or government policies.

To illustrate, we assume a community with the characteristics of the housing market in Taiwan. A model with mortgage credit only available to homeowners yields higher homeownership rates – and is more stable in the face of changes in other market characteristics – than can be achieved with other credit configurations. We also test the behaviors of households and investors when mortgage rates change relative to their respective cost of capital. When the mortgage rate is lower than the cost of capital of a household, the subsidized mortgage credit will increase homeownership rates if credit is granted only to homeowners but not to investors. However, if such a mortgage credit is also accessible to investors, this creates unique elasticities of the homeownership rate with respect to changes in the market-defining parameters.

This research is also of policy interest from the perspective of homeownership rate. Expanding or maintaining homeownership rates has been a primary social

goal in many countries. It is important, therefore, to understand how policy levers may affect economic incentives, and how those incentives then affect the balance of demand for ownership of the housing inventory by the competition between households against housing investors. Shifts in that balance directly impact the final homeownership rate. Our research suggests that policy choices must respect the simultaneous and competing nature between households and investors in the context of the overall market conditions.

One example of how policy must be cognizant of the competition between owner-occupiers and investors for housing stock is in major cities where artificial housing shortages have been created by high expected growth rates of house prices. Many large cities in China are good examples of this phenomenon, where housing units have not been purchased for occupancy or renting, but holding and flipping. Similar conditions are also observed in the developed countries, such as the US during the subprime boom years. Our model highlights how increases in expected growth rates in house prices reduce the reservation purchase prices for both households and investors in the rental market, thus stimulating both groups to invest in housing. Many researchers have argued that taxation is an important factor, particularly as a policy tool, in influencing housing tenure and homeownership rate. Herbert *et al.* (2014) summarize several studies that identify whether owning or renting is likely to be more financially beneficial and the circumstances while considering the tax effect. For instance, assuming that owners can take full advantage of tax benefits at a 28% marginal rate and 7% of rent-to-value ratio, Mills (1990) finds that a holding period of slightly longer than 7 years is necessary for preference to own. Following Mills (1990), Capone (1995) examines the financial value of homeownership for households in the 15% federal tax bracket and argues that rent-to-value ratios of 10% to 12% are more reasonable. Under this new assumption, owners only need to keep their homes for about 3 years for preference to own.

In addition to the difference in income tax rate, there are several other tax regulations for the parties involved in the housing market, *i.e.*, homeowners, landlords, and tenants. For example, property, capital gains and passive income taxes, tax deductions for rental expenses and mortgage interest, progressive tax rates, etc. Bourassa and Peng (2011) argue that the one of the factors behind the high homeownership rate in Taiwan is its low property tax rate. However, different combinations of housing taxation rules are implemented by governments in different regions. The large variation in housing tax policies and their potential impacts to the equilibrium of the housing market are very complicated to model and highly sensitive to region-specific taxation rules. For this paper, we focus on linking the ideology behind micro tenure choices with observations of macro homeownership rates. All of the analyses in this paper are conducted on a before-tax basis. Nevertheless, our modeling framework can be extended to analyze the potential impacts of various tax treatments on the bidding competition among households and investors.

A literature review is provided in the next section. Section 3 provides an introduction on our model to understand the microfoundations of homeownership rates with the use of the R/P ratio construct. In Section 4, we run sensitivity analyses to show the impacts of the parameter variables on the reservation prices of individual households for renting, reservation prices of investors for entering the rental housing market, and the final homeownership rate under such a competition. Section 5 discusses the results and the final section concludes.

2.   Background Literature Review

Research devoted to this important issue has mostly focused on the determinants of individual household tenure choice—owning or renting. We summarize the findings of past studies into six areas.

1) Downpayment requirements for obtaining mortgage loans have effective wealth constraints on the homeownership choice (Brueckner 1986; Linneman and Wachter 1989; Jones 1990; Bourassa 1995; Linneman et al. 1997; Wood et al. 2006).

2) Permanent income also affects the demand for housing investment by owner-occupiers. Researchers speak of the "tilt" effect of fixed rates and payment mortgages, whereby households have the incentive to take on larger monthly payments at the time of purchase, with the expectation that the burden will lighten as their income increases over time (Stevens, 1979; Haurin, 1991; Haurin et al.,1994; Gyourko and Linneman, 1996).

3) High expected capital gains in the housing market will motivate households to buy homes (Follain, 1982; Goodman, 1988; Dusansky and Koc, 2007). However, higher house-price volatility also means higher investment risk, and so the variance in house price growth can have offsetting effects on the tenure choice decision (Tuner 2003). The opposite effect is found if the variance in rental rate is high. In that case, household demand for ownership increases (Sinai and Souleles, 2005).

4) The high transaction costs of selling homes make homeownership more expensive for mobile households, thus reducing their demand for housing investment (Boehm, 1981; Zorn, 1988; Capone, 1995; Ioannides and Kan, 1996; Goodman, 2002). The decision to move is jointly determined with that of the housing tenure type (Kan, 2000).

5) Property tax is a burden that affects the cost of owning a house and, thus, ownership demand. While property taxes might not seen as a service provision, they have some deadweight component. Thus, if expected capital gains cannot compensate for the deadweight loss, then renting becomes preferable to owning (Hsien and Lin, 2000). Conversely, income tax subsidies to homeowners have a positive effect on the demand for owner-occupied housing (Rosen, 1979;

Rosen and Rosen, 1980; Rosen et al, 1984; Narwold and Sonstelie, 1994; Green and Vandell, 1999).

6)   Government housing-subsidy programs can play an important role in household tenure choice. The provision of government incentives to buy rather than rent will increase the demand of owner-occupants (Bourassa and Yin, 2006). In addition, if owning a house has intangible benefits, say, in social status, that too enhances demand and ownership rates (Grange and Pretorius, 2000).

In addition to the plethora of housing tenure-choice studies, there are other studies that directly discuss the determinants of community homeownership rates. For example, Eilbott and Binkowski (1985) find that household income, house price, household size, age distribution, and population change in metropolitan areas have very significant influences on household tenure choice, and such factors can, together, explain for 56% of the homeownership rate differences across cities. Coulson and Fisher (2002) use a Probit model to measure the marginal effect of the determinants of housing tenure choice, and then estimate the homeownership rates of different regions and states. They find that market factors have a higher explanatory power than individual household factors, and the relative cost of buying versus renting, population density, and population in central city areas are the most important factors. Painter and Redfearn (2002) find that interest rates influence both housing supply and housing tenure-type transitions, but a direct effect on the long-term homeownership rate cannot be substantiated. Thus, to promote homeownership rates, government intervention is more effective when directed at lowering down-payment requirements or taking on some credit (default) risk.

In summary, past studies have tended to investigate the determinants of housing tenure choice or homeownership rates separately but have not discussed how individual tenure choice affects the final homeownership rate in a community. We use a microeconomic foundation in which the homeownership rate is the cumulative result of individual tenure-choice decisions, which are themselves affected by many factors. The impact of a given set of values for the determinants of tenure choice will be reflected in the market equilibrium R/P ratio $(R/P)^*$ in the rental market. It is appropriate to focus on the rental market because the stock of houses owned by investors and the number of renter households must be equal. [1] Once that equilibrium is achieved, the homeownership rate is known.

In the long run, any gap between the market price for rentals, as specified in the R/P ratio, and the R/P ratio of the reservation price of ownership choice for the marginal renter household becomes the critical trigger for further housing-market adjustments. This paper develops the concept of an equilibrium R/P

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[1] One could include a normal housing vacancy rate in the rental market equilibrium. Without loss of generality, we assume that there are no vacant units.

ratio and the corresponding equilibrium homeownership rate to explain for the dynamics of housing market adjustments.

3.   Model Framework

In this section, we derive our model in a simple economy under three scenarios where households have: (3.1) no wealth constraint and no mortgage lending available, (3.2) wealth constraint but no mortgage lending available, and (3.3) wealth constraint and mortgage lending available. By equating the total cost of owning to that of renting a house, we derive an incentive indifference R/P ratio as follows.

3.1   Simple Economy with No Wealth Constraint and No Mortgage Lending

Imagine a simple economy where the number of housing units perfectly matches the number of households. There is an investor who is interested in buying housing units to rent to households. There are no taxes, and no mortgage lending is available. Households are differentiated solely by the length of their expected housing tenure, T_j. The total cost of owning a unit of housing in this economy includes the initial acquisition cost (AC), user-cost of maintenance during the tenure period (UC), and sale cost at the end of the tenure period (SC). The cash outflow of the initial acquisition cost (AC) at time $t = 0$ for the household j can be described as:

$$AC_j = -(1 + \delta_b)P_0 \tag{1}$$

where P_0 denotes initial price of a standard housing unit, and δ_b is transaction cost of buying, as a fraction of the housing price, including brokerage fees, insurance, title registration, etc.

During the tenure period in the house, the user-cost of maintenance (UC) can be specified as:

$$UC_{j,t} = \phi P_t = \phi P_0 e^{\pi t} \tag{2}$$

where ϕ is maintenance cost, as a fraction of the housing price, including hazard insurance premium, etc. π equals expected growth rate of house prices.

At the end of the tenure period, T_j, the homeowner will sell the property and recover the selling cash flow (SC) as: [2]

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[2] In many countries, homeowners will pay some taxes (*e.g.*, a capital gains tax) when they sell a house, but tax treatments are quite different across countries. For simplicity, we do not consider a capital gains tax in our analysis. Eliminating capital gains on owners and investors is equivalent to assuming a situation whereby investors and owner-

$$SC_j = (1 - \delta_s)P_{T_j} = (1 - \delta_s)P_0 e^{\pi T_j} \tag{3}$$

where δ_s is transaction cost of selling a house at time T_j, as a fraction of the house price.

The total cost of owning a house for T_j periods can be specified as:

$$
\begin{aligned}
CO_j &= AC_j + \int_0^{T_j} PV(UC_{j,t})dt - PV(SC_j) \\
&= (1 + \delta_b)P_0 + \int_0^{T_j} \phi P_0 e^{(\pi-y)t} dt - (1 - \delta_s)P_0 e^{(\pi-y)T_j}
\end{aligned}
\tag{4}
$$

where y denotes periodic discount rate for households. PV is present value function.

Note that the discount rate for households is their opportunity cost of capital. On the other hand, the total cost of renting the same house from an investor for T_j periods can be described as:

$$CR_j = \int_0^{T_j} PV(R_t)dt = \int_0^{T_j} R_0 e^{(\pi_r - y)t} dt \tag{5}$$

where R_0 is market rent of the housing unit at time 0, and π_r is expected growth rate of rent which is set to be equal to π in our simple economy.

Given the values of the set of input parameters $\{\delta_b, \delta_s, \phi, \pi, y, T_j\}$, households that are not wealth-constrained will make their housing tenure choice, i.e., owning or renting, depending on whichever is lower in cost. A household is indifferent only when $CO_j = CR_j$. By setting Equation (4) to equal to Equation (5), we solve the particular R/P ratio for each household j and define the indifference price in terms of R/P ratio, which is a function of the input parameter set:

$$(R_0/P_0)_j^* = f(\delta_b, \delta_s, \phi, \pi, y, T_j) \tag{6}$$

Note that the only input parameter specific to household j is the length of the tenure period, T_j. For all values of $(R_0/P_0) > (R_0/P_0)_j^*$, the household j will choose to buy a house rather than rent. Otherwise renting is the optimal tenure mode.[3] Equating Equations (4) to (5) gives :

$$CO_j = CR_j$$

or

$$(1 + \delta_b)P_0 + \int_0^{T_j} \phi P_0 e^{(\pi-y)t} dt - (1 - \delta_s)P_0 e^{(\pi-y)T_j} = \int_0^{T_j} R_0 e^{(\pi-y)t} dt$$

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occupiers both have options to avoid a capital gains tax. For investors, this would generally be through rolling-over gains into new real estate investments.
[3] We will introduce the effect of wealth constraints on this choice in a later section.

Solving the integral $\int_0^{T_j} e^{(\pi - y)t} dt = \dfrac{e^{(\pi - y)T_j} - 1}{\pi - y}$ and completing the algebraic derivation result in:

$$(R/P)_j^* = \frac{(\pi - y)(\delta_b + \alpha\delta_s) - (\alpha - 1)(\pi - y - \phi)}{\alpha - 1} \qquad (7)$$
$$\text{where } \alpha = e^{(\pi - y)T_j}$$

Note that we remove the 0 subscripts on R_0 and P_0 for ease of reading. Assuming risk-averse households, we impose the condition, $\pi - y < 0$ (or $0 < \alpha < 1$),[4] which yields the following series of relationships between the input parameters and reservation prices of households for renting:

$$\frac{\partial(R/P)_j^*}{\partial\delta_b} = \frac{\pi - y}{\alpha - 1} > 0, \quad \frac{\partial(R/P)_j^*}{\partial\delta_s} = \frac{\alpha(\pi - y)}{\alpha - 1} > 0,$$

$$\frac{\partial(R/P)_j^*}{\partial\phi} = 1, \quad \frac{\partial(R/P)_j^*}{\partial T_j} = \frac{-\alpha(\pi - y)^2(\delta_b + \delta_s)}{(\alpha - 1)^2} < 0,$$

$$\frac{\partial(R/P)_j^*}{\partial\pi} = \frac{[(\alpha - 1) - (\pi - y)\alpha T_j]\delta_b}{(\alpha - 1)^2}$$
$$+ \frac{[(\alpha - 1) - (\pi - y)T_j]\alpha\delta_s - (\alpha - 1)^2}{(\alpha - 1)^2}, \qquad (8)$$

$$\frac{\partial(R/P)_j^*}{\partial y} = -\frac{[(\alpha - 1) - (\pi - y)\alpha T_j]\delta_b}{(\alpha - 1)^2}$$
$$- \frac{[(\alpha - 1) - (\pi - y)T_j]\alpha\delta_s - (\alpha - 1)^2}{(\alpha - 1)^2}$$

Positive partial derivatives of indifference prices on $(\delta_b, \delta_s, \phi)$ confirm that increasing the values of the cost parameters will increase the R/P ratio of the reservation price above which households will choose owning over renting. Such an increase in the cost parameter reduces demand for owner-occupied housing and thus reduces the homeownership rate. On the other hand, the negative partial derivative on T_j means that increases in the expected tenure period reduce the relative cost of owning versus renting, and the maximum rent that a household j is willing to pay. That leads to a higher demand for owner occupancy and thus a higher ownership rate. Without loss of generality, we assume the expected tenure periods of a household follow a lognormal distribution.[5] Then, the demand curve for rental housing can be derived from

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[4] In a normal functioning economy, we would expect y > π. That is, risk-averse households will require the risk adjusted return on the risky housing asset to be higher than the expected growth rate of house prices.

[5] A lognormal distribution is a convenient tool because it assumes that households are randomly assigned to tenure periods via an underlying normal (Gaussian) distribution. The lognormal density function can also take on a variety of (unimodal) shapes.

Equation (7), which will follow the shape of a cumulative lognormal distribution function for T_j as shown in Figure 1, where Q denotes the demand for rental housing for the corresponding R/P ratio and Q_{all} is all of the houses available for renting.

Figure 1   Market Demand Curve for Rental Housing
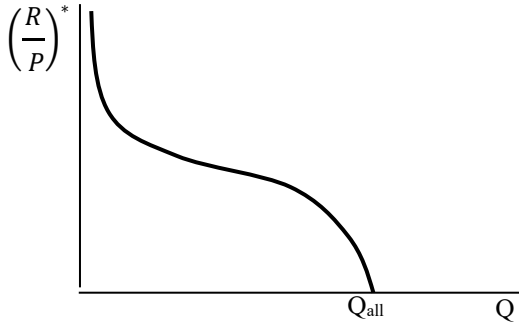
We now add to our simple economy a large number of homogenous housing investors who do not occupy housing units but are interested in purchasing them to rent out before selling for a capital gain. Housing investors require a rate of return, say $y + \psi$, where ψ is the required risk premium. Their cash flow from housing investments is the same as that of the homeowners, except that the investors receive rent as revenue during the holding period. Thus, net present value of an investor i from a housing investment, NPV_i is:

$$NPVi = -(1 + \delta_b)P_0 + \int_0^{T_i} (R_0 - \phi P_0)e^{(\pi - y - \psi)t} dt \qquad (9)$$
$$+ (1 - \delta_s)P_0 e^{(\pi - y - \psi)T_i}$$

In a competitive investment environment, each investor earns zero NPV. The R/P ratio for breaking even on the supply side of rental housing is found by setting Equation (9) to zero and solving for $(R/P)^*$:

$$(R/P)_i^* = f(\delta_b, \delta_s, \phi, \pi, y, \psi, T_i) \qquad (10)$$

where i denotes investor index, T_i is investor holding period, which is equal for all i.

We can get:

$$(R/P)_i^* = \frac{\gamma \left[\frac{(\beta - 1)\phi}{\gamma} + (1 + \delta_b) - \beta(1 - \delta_s) \right]}{\beta - 1}; \qquad (11)$$
$$\text{where } \beta = e^{(\pi - y - \psi)T_i}, \gamma = \pi - y - \psi$$

Assuming risk-adverse investors, we have $\gamma < 0$ (or $0 < \beta < 1$) and obtain the following:

$$\frac{\partial (R/P)_i^*}{\partial \delta_b} = \frac{\gamma}{\beta - 1} > 0, \quad \frac{\partial (R/P)_i^*}{\partial \delta_s} = \frac{\beta\gamma}{\beta - 1} > 0,$$

$$\frac{\partial (R/P)_i^*}{\partial \phi} = 1, \quad \frac{\partial (R/P)_i^*}{\partial \psi} = 1,$$

$$\frac{\partial (R/P)_i^*}{\partial T_i} = \frac{-\beta\gamma^2(\delta_b + \delta_s)}{(\beta - 1)^2} < 0, \tag{12}$$

$$\frac{\partial (R/P)_i^*}{\partial \pi} = \frac{[\beta(1 - \gamma T_i) - 1]\delta_b}{(\beta - 1)^2}$$
$$+ \frac{\beta\delta_s[(\beta - 1) - \gamma T_i] - (\beta - 1)^2}{(\beta - 1)^2} = \frac{\partial (R/P)_i^*}{\partial y}$$

The positive impacts of $(\delta_b, \delta_s, \phi, \psi)$ on $(R/P)^*$ confirm that increasing the value of the cost parameters will increase the minimum rent required by investors. In contrast, the equilibrium R/P ratio for investors, $(R/P)_i^*$ decreases as T_i increases. By assuming all investors are homogenous and perfect competitors, the R/P ratio for breaking even is a constant, thus implying a horizontal supply curve for rental housing. The juxtaposition of supply and demand for rental housing then resembles the one illustrated in Figure 2.
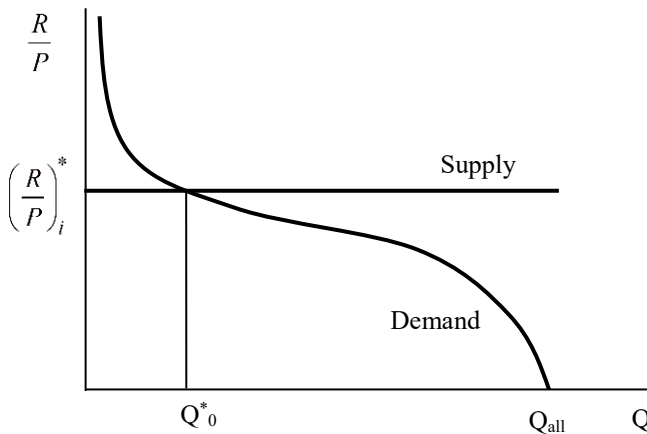
Figure 2   Supply and Demand for Rental Housing

Figure 2 shows that when the rental housing market is highly competitive, the equilibrium R/P ratio will be completely determined by the breakeven value of the investors, $(R/P)_i^*$. Typically, T_i is selected by the investor to maximize the internal rate of return (IRR) of the investment project. Based on homogenous

expectations and competitive market assumptions, all of the investors will have the same T_i. This T_i will set the R/P ratio for supply. Similarly, the household side needs to go through an R/P ratio conversion analysis based on their rent vs. buy decisions to determine the specific breakeven T_j for which the household is indifferent between renting and buying. As a result, all households with personal $(R/P)_j^*$ ratios greater than the indifference point, $(R/P)_i^*$, will self-select renting over buying, where Q_0^* in Figure 2 is the number of rental households. They represent those with the shortest expected tenure periods. Other households with reservation prices for renting $(R/P)_j^*$ that are less than $(R/P)_i^*$ will find that owning is more financially advantageous and self-select owning a house instead of renting when their expected tenures, T_j, are longer than their breakeven tenure as implied by $(R/P)_i^*$, T_m^*. Once this marginal tenure requirement for homeownership rate (denoted as Ω) is determined, the equilibrium homeownership of this community is identified by a cumulative lognormal distribution:

$$\Omega = 1 - F\left(T_j^* \leq T_m^*\right) = 1 - Q_0^*/Q_{all} \tag{13}$$

3.2   Wealth Constraint but No Mortgage Lending Available

In this section, we relax the assumption of a no-wealth-constraint, which implies that all households are financially capable of paying P_0 to purchase a house if they choose to do so. These wealth constrained households can only be renters because their wealth is less than P_0, which means that owning is an infeasible option, unless borrowing becomes available. The wealth constraint then shifts the demand curve for rental housing to the right as shown in Figure 3, where Q^c is the wealth-constrained population.[6] Similarly, it can be argued that some investors may also be subject to a wealth constraint. However, under the assumption of a large number of potential investors and a competitive market, only the investors who are not subject to wealth constraints are likely to win in a competitive housing bidding market. Under wealth constraints, the number of rental households will increase from Q_0^* to Q_1^* in Figure 3.

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[6] In our simple economy, these households may have some positive wealth, but just less than $P^*(1+\delta_b)$.

Figure 3   Supply and Demand for Rental Housing with Wealth Constraint
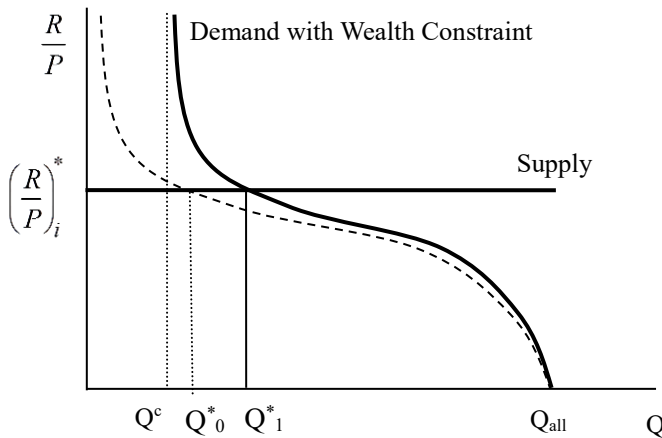
According to the expected tenure of a household in the house, the new demand curve resembles a cumulative lognormal function, shaped by the values of the parameters and y. In this modified economy, the equilibrium homeownership rate declines to:

$$\Omega = \frac{(Q_{all} - Q^c)[1 - F(T_j^* \leq T_m^*)]}{Q_{all}} = 1 - \frac{Q_1^*}{Q_{all}} \qquad (14)$$

Equation (14) is derived from Equation (13). In Equation (13), one minus the ratio of the rental households (the number of rental households (Q_0^*) divided by total households) is the homeownership rate. However, some households that prefer to own may be constrained by the down payment requirement due to wealth constraint, so the actual rental households will be Q_0^* plus the number of wealth constrained households, and the sum is Q_1^*.

3.3   Wealth Constraint with Mortgage Lending Available

In this section, mortgage lending is made available for the wealth constrained population. When they decide to take a loan, borrowers can choose the amount of credit to borrow, which is denoted as the initial loan-to-value (LTV) ratio of the mortgage. Borrowers only need to have enough assets to pay for the rest of the purchase price, $(1 - \omega)P_H$, as the down payment. Given that mortgage credits are available, we derive the equilibrium R/P ratio $(R/P)^*$ and its partial derivatives with respect to the key parameters mentioned above from two situations: (3.3.1) mortgage lending available only for households, and (3.3.2) mortgage lending available for both households and investors.

3.3.1   Mortgage Lending Available only for Households

Once mortgage lending is available, the residential mortgage market mitigates the potential suboptimal allocation of resources caused by household wealth constraints, thus permitting the ownership decision to be based more on permanent income than on initial wealth. To show the effects of mortgage financing, we permit households to borrow a certain fraction of the initial purchase price, P_0, at a certain interest rate, r. To simplify the situation, we assume that all mortgages are interest-only and due upon sale. The addition of this mortgage option changes the function for the cost of the household to own from Equation (4) to:

$$CO_j^M = -(\delta_b + (1-\omega))P_0$$
$$-\int_0^{T_j}(\phi P_0 e^{(\pi-y)t} + \omega r P_0 e^{-yt})dt \qquad (15)$$
$$+(1-\delta_s)P_0 e^{(\pi-y)T_j} - \omega P_0 e^{-yT_j}$$

where ω is initial LTV ratio of the mortgage; $(1-\omega)$ is down payment rate. The last term in Equation (15) represents the loan repayment in time T_j.

The indifference R/P ratio for this mortgaged household becomes:

$$(R/P)_j^* = f(\delta_b, \delta_s, \omega, \phi, \pi, y, r, T_j) \qquad (16)$$

or

$$(R/P)_j^* = \frac{(\pi-y)\left(\dfrac{(\alpha-1)\phi}{\pi-y} + \dfrac{(1-e^{-yT_j})\omega r}{y}\right)}{\alpha-1}$$
$$+ \frac{(\pi-y)\left((1+\delta_b-\omega+e^{-yT_j}\omega) - \alpha(1-\delta_s)\right)}{\alpha-1}; \qquad (17)$$
$$e^{(\pi-y)T_j} = \alpha$$

Again, in a normal economy, $\pi - y < 0$ and $0 < \alpha < 1$. We have the following relationships between the input parameter values and reservation prices of the borrower:

$$\frac{\partial (R/P)^*_j}{\partial \delta_b} = \frac{\pi - y}{\alpha - 1} > 0, \quad \frac{\partial (R/P)^*_j}{\partial \delta_s} = \frac{\alpha(\pi - y)}{\alpha - 1} > 0,$$

$$\frac{\partial (R/P)^*_j}{\partial r} = \frac{(1 - e^{-yT_j})(\pi - y)\omega}{(\alpha - 1)y} > 0, \quad \frac{\partial (R/P)^*_j}{\partial \phi} = 1,$$

$$\frac{\partial (R/P)^*_j}{\partial \omega} = \frac{(1 - e^{-yT_j})(\pi - y)(r - y)}{(\alpha - 1)y} > 0, \text{ if and only if } y < r$$

$$\frac{\partial (R/P)^*_j}{\partial \pi}$$
$$= \frac{(\alpha - 1)[(y - r)\omega + e^{yT_j}(r\omega - y(\alpha - 1 + \omega) + y\delta_b + \alpha y\delta_s)]}{e^{yT_j}(\alpha - 1)^2 y}$$
$$+ \frac{\alpha T_j(\pi - y)[(r - y)\omega - e^{yT_j}((r - y)\omega + y(\delta_b + \delta_s))]}{e^{yT_j}(\alpha - 1)^2 y} \tag{18}$$

$$\frac{\partial (R/P)^*_j}{\partial y}$$
$$= \frac{(\pi - y)yT_j[(y - r)\omega + \alpha e^{yT_j}((r - y)\omega + y(\delta_b + \delta_s))]}{e^{yT_j}(\alpha - 1)^2 y^2}$$
$$+ \frac{(\alpha - 1)[(r\pi - y^2)\omega - e^{yT_j}(\pi r\omega - y^2(\alpha - 1 + \omega) + y^2(\delta_b + \alpha\delta_s))]}{e^{yT_j}(\alpha - 1)^2 y^2}$$

$$\frac{\partial (R/P)^*_j}{\partial T_j}$$
$$= \frac{(r - y)(y - \alpha\pi)\omega + e^{\pi T_j}\alpha(\pi - y)((r - y)\omega + y(\delta_b + \delta_s))}{e^{yT_j}(y - \pi)^{-1}(\alpha - 1)^2 y}$$

The positive impacts of $(\delta_b, \delta_s, r, \phi)$ on $(R/P)^*$ confirm that increasing the value of the cost parameters will increase the reservation prices of renting, and may subsequently lead to a lower homeownership rate as a result. As lenders permit higher initial LTV ratios, ω, fewer household will be constrained by initial wealth, and thus more will have the option of homeownership. If no down payments are required and the mortgage interest rate is equal to discount rate of the household, the mortgage loan will perfectly offset the wealth constraint and push the demand curve to its original position prior to the introduction of wealth constraints.

However, if the mortgage interest rate (r) is higher than the discount rate of the household (y), the demand curve for rental housing will shift upward due to the higher cost of owning. This would cause the homeownership rate to fall between the constrained and unconstrained levels. Note that the homeownership rate with mortgage availability can never fall below that with no mortgage option. That natural boundary exists because taking out a mortgage is a real option to the household. If the mortgage is too expensive, borrowers

can always be better off by refusing to take the mortgage. The situation would be identical to the case where a mortgage is not feasible with wealth constraint.

On the other hand, if the mortgage rate (r) is lower than discount rate of the household (y), households will find it less expensive to buy a house with leverage, which may result in reducing the rental demand, shifting the demand curve downward, and increasing the equilibrium homeownership rate as shown in Figure 4. The number of rental households will decrease from $Q_1{}^*$ to $Q_2{}^*$. That is, $\partial\Omega/\partial Q^c < 0$, $\partial\Omega/\partial\omega > 0$ and $\partial\Omega/\partial r < 0$. Hence, lower interest rates or a larger LTV ratio may help to realize the policy aim of increasing the homeownership rate only if the mortgage rate is lower than the discount rate of the household.

Figure 4   Supply and Demand for Rental Housing with Mortgage Availability
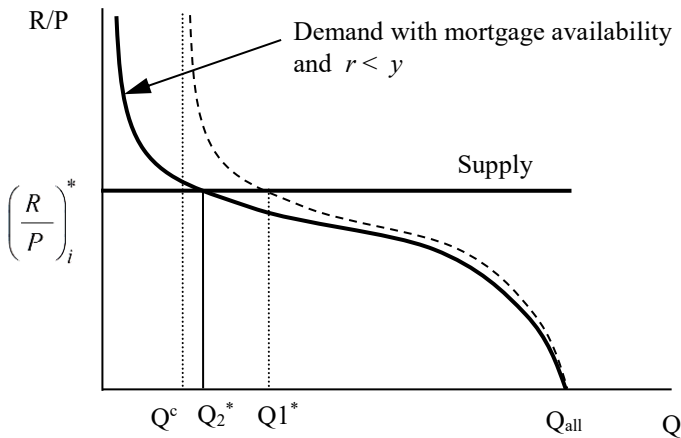
3.3.2   Mortgage Lending Available for both Households and Investors

When investors can also use mortgage leverage, their *NPV* in Equation (9) becomes:

$$NPV = -\left(\delta_b + (1 - \omega)\right)P_0$$
$$+ \int_0^{T_i} \left[(R_0 - \phi P_0)e^{(\pi - y - \psi)t} - \omega r P_0 e^{-(y+\psi)t}\right]dt \qquad (19)$$
$$+ (1 - \delta_s)P_0 e^{(\pi - y - \psi)T_i} - \omega P_0 e^{-(y+\psi)T_i}$$

Holding other conditions unchanged, $(R/P)_i^*$ can be expressed as follows:

$$(R/P)^*_i = \frac{\gamma\left(e^{-(y+\psi)T_i} + \frac{(\beta-1)\phi}{\gamma} + \frac{\left(1 - e^{-(y+\psi)T_i}\right)\omega r}{y + \psi}\right)}{\beta - 1}$$
$$+ \frac{\gamma\left(\delta_b - \beta(1-\delta_s) + (1-\omega)\right)}{\beta - 1} \tag{20}$$

where $\beta = e^{(\pi - y - \psi)T_i}$ and $\gamma = \pi - y - \psi$

Assuming $\pi < y + \psi$, then $\gamma < 0$ ($0 < \beta < 1$) and the following relationships between the input parameters and the equilibrium R/P ratio for investors hold:

$$\frac{\partial (R/P)^*_i}{\partial \delta_b} = \frac{\gamma}{\beta - 1} > 0, \quad \frac{\partial (R/P)^*_i}{\partial \delta_s} = \frac{\beta\gamma}{\beta - 1} > 0, \quad \frac{\partial (R/P)^*_i}{\partial \phi} = 1,$$

$$\frac{\partial (R/P)^*_i}{\partial r} = \frac{2\left(1 - e^{-(y+\psi)T_i}\right)\gamma\omega}{(\beta - 1)(y + \psi)}$$
$$+ \frac{\left(\delta_b + (1-\omega) + e^{-(y+\psi)T_i} + (\delta_s - 1)\beta\right)(\gamma + \psi)}{(\beta - 1)(y + \psi)} > 0,$$

$$\frac{\partial (R/P)^*_i}{\partial \pi} = \frac{(\beta - 1 - \beta\gamma T_i)e^{-(y+\psi)T_i}(y + \psi - \omega r)}{(\beta - 1)^2(y + \psi)}$$
$$+ \frac{(\beta - 1 - \beta\gamma T_i)(y + \psi)(\delta_b + \beta\delta_s - \omega) + (\beta - 1 - \beta\gamma T_i)\omega r}{(\beta - 1)^2(y + \psi)}$$
$$+ \frac{(1 - \beta)(\beta - 1 - \beta\gamma T_i\delta_s)(y + \psi)}{(\beta - 1)^2(y + \psi)},$$

$$\frac{\partial (R/P)^*_i}{\partial y} = \frac{e^{-(y+\psi)T_i} + \frac{\left(1 - e^{-(y+\psi)T_i}\right)\omega r}{y + \psi} + \delta_b - \beta(1-\delta_s) + 1 - \omega}{(1 - \beta + \gamma T_i\beta)^{-1}(\beta - 1)^2}$$
$$+ \frac{\gamma}{\beta - 1}\left[-T_i e^{-(y+\psi)T_i} + \frac{\left((y+\psi)T_i e^{-(y+\psi)T_i} - 1 + e^{-(y+\psi)T_i}\right)\omega r}{(y + \omega)^2}\right] \tag{21}$$
$$+ \frac{\gamma}{\beta - 1}[T_i\beta(1 - \delta_s)],$$

$$\frac{\partial (R/P)^*_i}{\partial \psi} = \frac{e^{-(y+\psi)T_i} + \frac{\left(1 - e^{-(y+\psi)T_i}\right)\omega r}{y + \psi} + \delta_b - \beta(1-\delta_s) + 1 - \omega}{(1 - \beta + \gamma T_i\beta)^{-1}(\beta - 1)^2}$$
$$+ \frac{\gamma}{\beta - 1}\Bigg[-T_i e^{-(y+\psi)T_i}$$
$$+ \frac{\left((y+\psi)T_i e^{-(y+\psi)T_i} - 1 + e^{-(y+\psi)T_i}\right)\omega r}{(y + \omega)^2}\Bigg]$$
$$+ \frac{\gamma}{\beta - 1}[T_i\beta(1 - \delta_s)] + 1$$

$$\frac{\partial (R/P)_i^*}{\partial T_i} = \frac{e^{-(y+\psi)T_i}\gamma(\omega\gamma - y - \psi)}{\beta - 1} - \frac{\gamma^2\beta(1 - \delta_s)}{\beta - 1}$$
$$- \frac{\gamma^2\beta}{(\beta - 1)^2}\left[e^{-(y+\psi)T_i} + \delta_b - \beta(1 - \delta_s) + (1 - \omega)\right]$$
$$- \frac{\gamma^2\beta\left(1 - e^{-(y+\psi)T_i}\right)\omega r}{(\beta - 1)^2(y + \psi)}$$

The positive impacts of $(\delta_b, \delta_s, \phi, r)$ on $(R/P)^*$ confirm that increasing the value of the cost parameters will increase the minimum rent required by investors. This means that investing in housing becomes less attractive than other alternative investment vehicles as the transaction cost in housing increases. When mortgage rate $r < (y + \psi)$, using a mortgage will improve the NPV from the project, pushing down the breakeven $(R/P)_i^*$ ratio. This is generally referred to as "positive leverage" in the real estate investment industry. This reduction in the marginal cost of the rental housing supply is represented by a downward shift of the rental supply curve from $(R/P)_i^*$ to $(R/P)_i^{**}$, and the number of rental households will increase from Q_2^* to Q_3^* as seen in Figure 5. This result will lead to an interesting policy implication: when subsidized mortgage lending is available for both households and investors, the homeownership rate may actually decrease as the allowable LTV increases as long as the mortgage rate is less than the opportunity cost of capital of the investor. We further investigate this implication through simulations in the following sections.

Figure 5   Supply and Demand for Rental Housing with Mortgage Availability
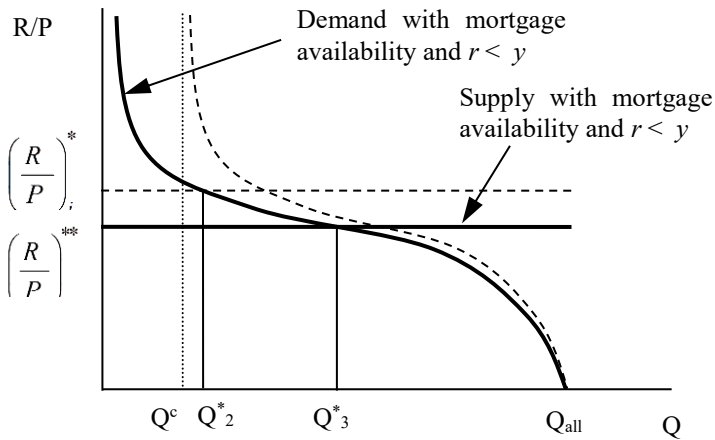
4.   Sensitivity Analysis

In this section, we examine the impacts of the explanatory variables on $(R/P)_j^*$, $(R/P)_i^*$ and homeownership rate (Ω) through sensitivity analyses by using a set of parameter values from typical housing markets in Taiwan as a base case. In a competitive market, the costs of capital are determined by the general capital market. Individual parties have little influential power and, thus, passively accept these exogenously determined discount rates. The distinctive feature of our parameter calibration is that the order of the cost of capital from the highest to lowest is investors, banks (mortgage rate), and households, or $y + \psi > r > y$. One possible explanation is as follows. In most capital markets, the mortgage rate is among the lowest lending rates offered by banks. However, the rate still needs to be at least higher than the risk-free rate to prevent arbitrage opportunities. Typically, banks have the lowest cost of capital among institutional investors. The deposit rate is the cost of borrowing for banks, which tends to be lower than the yield on any corporate bond. For investors, their opportunity cost of capital may be against stock market returns of comparable volatility which generally offer higher rates of return than risk free rates. Thus, the rental housing market needs to offer equivalent returns to be attractive for these investors. For households, their objective is to minimize the present value of future expenses that result from occupying a specific house unit. Hence, the discount rate for future cash outflows could be less than or equal to the risk free rate. That is why the discount rate for households could be the lowest among the three groups. The parameter values of our base case are listed in Table 1.

Table 1   Parameter Values for the Housing Market in Taipei

Symbol	Parameter	Initial Value (TWD)*
P_0	Price of standard house (100 m² /unit)	$10,000,000
R_0	Annual rental price of standard house	$ 300,000
δ_b	Transaction cost for buying a house	2%
δ_s	Transaction cost for selling a house	4%
ϕ	Maintenance costs, including hazard insurance	4%
π	Expected growth rate of housing prices and rent	0%
y	Discount rate for households	5%
ψ	Required risk premium by investors	2%
T_j	Expected household tenure period (in years)	7
T_i	Expected rental-investment period (in years)	12
ω	Loan to value ratio	70%
r	Mortgage interest rate	6%

Note: * TWD is New Taiwan Dollar; 1 TWD $\approx$ 0.034 USD.

Based on Table 1, we calculate the partial derivatives for and in both the simple and modified economies with mortgage credit. The results are shown in Table

2. As expected, the signs for the impacts of T_j and ϕ on $(R/P)^*$ are the same. The partial derivatives of both the demand and supply for rentals with respect to π are negative in both economies. That means increasing the expected rate of growth of housing price increases the demand for house purchase by both households and investors. Note that the signs of the partial derivatives of $(R/P)_j^*$ with respect to y and ω are both positive. Thus, as the household cost of capital (y) increases, demand for rental housing also increases. However, increasing the LTV ratio (ω) also increases rental demand since the mortgage rate (r) is higher than the household cost of capital (y).

For households, the negative impact of T_j on an equilibrium R/P ratio means that increasing the expected tenures in the house will increase the attractiveness of owning. For investors, we find that the signs of partial derivatives with respect to T_i and π are negative and those for y and ψ are positive in both economies. Furthermore, the sign for ω is negative since the cost of capital of the investors ($y + \psi$) is higher than the mortgage rate (r) in our base case setting.

Table 2   Impact of Parameters on Demand and Supply of Rental Housing

| Parameter (x's) | Households: $\dfrac{\partial (R/P)_j^*}{\partial x}$ | | | | Investors: $\dfrac{\partial (R/P)_i^*}{\partial x}$ | | | |
| | Without Mortgage Credit | | With Mortgage Credit | | Without Mortgage Credit | | With Mortgage Credit | |
	Expected sign	Simulated Value	Expected sign	Simulated Value	Expected sign	Simulated Value	Expected sign	Simulated Value
δ_b	+	0.1693	+	0.1693	+	0.0929	+	0.0929
δ_s	+	0.1193	+	0.1193	+	0.0229	+	0.0229
ϕ	+	1	+	1	+	1	+	1
π	?	-0.9935	?	-1.0166	?	-1.0032	?	-1.0445
y	?	0.9935	?	0.2935	?	1.0032	?	0.1892
ψ					?	1.0032	?	1.1892
T	—	-0.0012	?	-0.0012	—	-0.0001	?	-0.0024
ω			?	0.01				-0.01
r			+	0.7			+	0.7

4.1   Impacts of Expected Growth Rate of House Prices on Homeownership Rates

In this section, we examine the impacts of expected growth rate of house prices on homeownership rates in two situations: a (4.1.1) simple economy without mortgage availability, and a (4.1.2) simple economy with mortgage availability.

4.1.1   Simple Economy without Mortgage Availability

In Table 2, we find that the expected growth rate of house prices (π) is negatively related to (R/P)* for both households and investors, which means that the demand and supply curve for rentals will move downward simultaneously as π increases in a simple economy with or without mortgage availability. Thus, the market clearing R/P ratio will unambiguously fall, but the final direction of change in the homeownership rate is indeterminate. Given the parameter values based on the housing market in Taiwan, increasing the expected housing appreciation rate will motivate households (investors) to reduce down their maximal (minimal) rental price that they are willing to pay (accept). The reaction of households to a higher expected growth rate of housing prices will normally cause a higher homeownership rate, but the reaction of the investors will have the opposite effect. As indicated in Table 2, the impact on the demand side (households) is -0.9935 which is not significantly different from that on the supply side (investors), -1.0032. Whether homeownership rate increases or declines will depend on the shape of the demand curve, *i.e.,* the distribution of the expected tenure among households.

In Table 3, we show the simulated outcomes for the homeownership rate (Ω), equilibrium $(R/P)^*$ ratio, and marginal housing tenure period (T_m^*) across four growth rates of house prices (-5%, 0%, 5.5%, and 10%). In a simple economy without mortgage availability, the homeownership rate is insensitive to large swings (from -5% to 10%) in expected house-price growth: there is just a slight decrease in Ω, from 93.9% to 93.3%. The marginal tenure period (T_m^*) is also insensitive to the changes in the expected growth rate of house prices (π). However, the market R/P ratio determined by investors changes dramatically from 16.46% as $\pi = 5\%$ to 1.54% as $\pi = 10\%$. As can be inferred from Table 2, some investors even choose to keep the unit vacant and speculate further for capital gains. The fact that these units are kept vacant effectively reduces the housing supply in the market, thus leading to an artificial housing shortage that further supports both future house price growth and rental prices. As a result, homeownership rates may fall substantially even though high house-price growth will otherwise favor more homeownership.

4.1.2   Simple Economy with Mortgage Availability

As indicated in Table 2, there is a negative impact of the expected growth rate of house prices (π) on (R/P)* for both households and investors; the impact on the demand side (households) is -1.0166 which is not significantly different from that on the supply side (investors) and -1.0445 in our base case for a simple economy with mortgage availability. Table 3 shows that an increase in π has a slightly negative impact on homeownership rate (Ω) when mortgage credits are only available to households as all homebuyers are required to use mortgage

credit as much as possible. However, the homeownership rate drops substantially for a given level of π, as the mandated LTV ratio for households increases because we assume that the discount rate of the household (y) is lower than the mortgage interest rate (r) in our simple economy. For example, when $\pi = 5.5\%$, the homeownership rate (Ω) decreases from 93.3% to 86.3% as the LTV ratio changes from zero to 70%.

When mortgage credit is available for both households and investors, the expected growth rate of house prices matters the most. It is worth noting that given π, the homeownership rate decreases significantly when the LTV ratio available for investors increases. For instance, when $\pi = 5.5\%$, Ω decreases from 86.3% to 74.1% as the mortgage credit available for investors changes from zero to 70%. Besides, the mortgage availability of investors even plays a decisive role while facing economic stagnation or downturns. In Table 3, we find that when $\pi \leq 0$, the homeownership rate decreases to around 46% which is only half of the rate if no mortgage is available for both the households and investors. Meanwhile, the marginal tenure period (T_m^*) is three times as long as the previous one, from 2.43 to 7.51 years. These results are expected because we assume that the cost of capital of the investors ($y + \psi$) is higher than the mortgage interest rate (r), and investors take advantage of financial leverage as much as they can once they have access to cheaper capital. As a result, the marginal cost of the rental housing supply declines significantly and thereby contributes to a lower equilibrium R/P ratio. In this example, $(R/P)^*$ drops to only 1.14%, which suggests that rental income becomes almost negligible when the expected house price becomes very high.

Actually, when the growth rate of house prices further increases, the equilibrium R/P ratio would fall to less than zero. This is an environment where investors can choose to keep the unit vacant and try to profit from "flipping" the property for capital gain. The fact that these units are kept vacant effectively reduces the housing supply in the market, thus leading to an artificial housing shortage that further boosts both future house price growth and rental prices. As a result, homeownership rates may fall substantially even though the high house-price growth will otherwise favor more homeownership. A high vacancy rate together with housing shortage will be simultaneously observed in such extreme market environments, as was observed in some of the fast growing housing markets, such as Beijing and Shanghai, as well as Las Vegas during the early 2000s global housing boom.

4.2   Impacts of Mortgage Rates on the Homeownership Rate

In the previous sections, we assume that the order of the cost of capital utilized on housing tenure decision among households (y), banks (r), and investors ($y + \psi$) is $y < r < (y + \psi)$ in our base case scenario. Facing wealth constraint, households and investors will use up all available mortgage credits regardless

of the percentage of the mortgage rate. In this section, we examine the impact of mortgage rates on homeownership rate in an alternative scenario where $r < y < (y + \psi)$. This captures the impact of certain government programs that subsidize the mortgage rates. In Table 4, we show the simulation outcomes for homeownership rate (Ω), equilibrium R/P ratio, and marginal housing tenure period (T_m^*) when the mortgage interest rate (r) is less or more than the discount rate of the household (y), $r = 4\%$ or 6%, respectively.

We find in this alternative scenario that: $r < y < (y + \psi)$, in which the mortgage rate (r) is the lowest among the three groups ($r = 4\%$) and unlike previous findings, Ω increases from 93.5% to 95.2% as the LTV ratio that is only available for households increase from zero to 70%. When mortgage credits of LTV ratio =70% are available for all homebuyers, Ω drops substantially from 95.2% to 64.1%. When the mortgage rates decrease from 6% to 4%, and switching the base case scenario to a subsidized mortgage environment, Ω increases from 46.0% to 64.1% and the marginal tenure period (T_m^*) drops from 7.51 to 5.45 years.

5.   Results and Discussion

In Tables 3 and 4, we can observe that the homeownership rates in our simple economy are very sensitive to the expected growth rates of house prices and mortgage interest rates if mortgage credits are available for both households and investors. These results reflect the long-run equilibrium dynamics after a permanent change in some of the input parameters. The values of the input parameters in Table 1 can be considered to be at the long run equilibrium level.

Table 3   Sensitivity of the Homeownership Rate (Ω), $(R/P)^*$, and T_m^* to Changes in π

Annual growth rate of house prices, π	Implied relationships among relevant interest rates	Credit Scenario (Household LTV ratio, Investor LTV ratio)		
		Outcome type	(1) (0%, 0%)	(2) (70%, 0%)	(3) (70%, 70%)
-5%	$\pi < 0 < y < r$	Ω	93.9%	86.3%	46.2%
		R/P*	0.1646	0.1646	0.1557
		T_m^*	2.38	3.27	7.49
0%	$(0 = \pi) < y$ $< r$	Ω	93.5%	85.7%	46.0%
		R/P*	0.1146	0.1146	0.1046
		T_m^*	2.43	3.34	7.51
5.5%	$y < \pi < r$ $< (y + \psi)$	Ω	93.3%	86.3%	74.1%
		R/P*	0.0599	0.0599	0.05469
		T_m^*	2.45	3.28	4.47
10%	$y < r$ $< (y + \psi) < \pi$	Ω	93.3%	87.0%	74.3%
		R/P*	0.0154	0.0154	0.0114
		T_m^*	2.45	3.20	4.45

Table 4   **Sensitivity of the Homeownership Rate (Ω), $(R/P)^*$, and T_m^* to Changes in r**

Mortgage Interest Rate	Implied relationships among relevant interest rates	Credit Scenario (Household LTV ratio, Investor LTV ratio)			
		Outcome type	(1) (0%, 0%)	(2) (70%, 0%)	(3) (70%, 70%)
4%	$r < y$ $< (y + \psi)$	Ω	93.5%	95.2%	64.1%
		R/P*	0.1146	0.1146	0.0936
		T_m^*	2.43	2.24	5.45
6%	$y < r$ $< (y + \psi)$	Ω	93.5%	85.7%	46.0%
		R/P*	0.1146	0.1146	0.1046
		T_m^*	2.43	3.34	7.51

Another reason for the elasticities shown in Tables 3 and 4 is the set of assumptions made about our economic agents. We assume that the investors are identical and operate in a perfectly competitive market. More importantly, we assume that our households make housing tenure choices according to their expected tenure periods, where the distribution of those periods can be modeled as lognormal. If the distribution of the expected holding periods for households is other than lognormal, or if the parameters of the lognormal density function differ from those that we have used, the corresponding homeownership rates may change with the implied demand curve. A different set of the parameters of the lognormal distribution will change the shape of the demand curve and subsequently change the elasticity of homeownership rates with respect to other model parameters. Thus, matching the tenure distribution to an actual population is important for policy analysis with our model. Our simulation results are summarized in Table 4 for the potential impact of a government subsidized low rate mortgage program that offers S_L: $r < y < (y + \psi)$, against the normal mortgages in the base case scenario S_B: $y < r < (y + \psi)$ as follows.

In our base case scenario (S_B) where $y < r < (y + \psi)$, the mortgage rate (r) is in between the cost of capital for the households (y) and investors ($y + \psi$). We find that increasing the value of the cost parameters (δ_b, δ_s, ϕ) will increase the reservation prices of renting above which households will choose owning over renting. Such increases in the cost parameters reduce demand for owner-occupied housing and thus reduce the homeownership rate. One may expect that increasing the mortgage size may increase the affordability of households and thereby the homeownership rate, but our simulation results in Table 3 show otherwise. The homeownership rate may decrease as the LTV ratio increases as long as the mortgage rate is less than the opportunity cost of capital of the investors. The amount of the decrease in the homeownership rate partially depends on the market outlook: the percentage is much larger when the housing market outlook is pessimistic, which is when the expected growth rate of housing prices is less than or equal to zero.

The partial derivatives of both the demand and supply for rentals with respect to the expected growth rate of house prices (π) are negative, which means that increasing π increases the breakeven R/P ratio for home purchases made by both the households and investors. However, we find that the homeownership rate is insensitive to the expected growth rate of house prices regardless of the market outlook. The positive impact of π on the homeownership rate is only evident when a regime switch occurs. Whether the homeownership rate will go up (down) eventually depends if the magnitude of the impacts of the above-mentioned parameters on the rental demand (supply) side is higher than the other.

In the scenario with a subsidized mortgage program (S_L) where $r < y < (y + \psi)$, the mortgage rate (r) is the lowest among the three groups. We find that when the mortgage rates decrease, the homeownership rate increases and the marginal tenure period in the house declines. However, increasing the LTV ratio substantially decreases the homeownership rate. If mortgage loans are universally offered to all economic agents, households and investors, borrowing can help wealth-constrained households to buy housing, but will also encourage investment in rental housing. Investors take advantage of "positive leverage" as much as they can once they have access to the cheaper (subsidized) mortgage credit. The positive leverage on the investor side can lead to a lower homeownership rate. As long as the required rate of return for investors is greater than the cost of capital of households, investors can obtain more benefits through positive leverage than households.

If the mortgage rate is in between the cost of capital for the households (y) and investors ($y + \psi$) as in the base case scenario, the households would want to minimize the positive leverage while investors prefer to maximize it. As a result, if the government wants to increase the homeownership rate through mortgage subsidies, those subsidies must be only available for owner-occupied units. However, it is very challenging to differentiate homeowners from investors. In the US, it is not uncommon for a household to own two housing units: one for primary residence and the other for a vacation home or investment purposes. During housing booms, many households become investors by purchasing a second home to profit from the capital gains. In sum, subsidized mortgage interest rates or higher LTV ratio mortgage programs will render homeownership more affordable, but it is critical to ensure the government subsidies are not used by housing investors as a means to speculate or flip housing.

6.   Conclusions

Previous studies have separately investigated the determinants of housing tenure choices and homeownership rates, but have not discussed the connection between the two issues. In this paper, we develop a theoretical model for determining the allocation of housing units available in a housing market. The fixed number of physical housing units will always be occupied by the same fixed group of local households, either as renters or homeowners. The process that determines whether a housing unit is owned or for rent is the competition between households and investors in bidding on the housing inventory. When a household is willing to pay a higher price than investors, the unit is owner occupied. Otherwise, it becomes a rental unit. By assuming a large number of homogenous investors, the equilibrium R/P ratio is determined by the breakeven ratio of investors when housing is considered as an alternative to other investment vehicles. Under this equilibrium R/P ratio, households self-sort themselves into owners or renters according to their indifference R/P ratio, which is captured by their expected tenure in this paper.

Based on this configuration, we derive the impacts of the key parameters, such as expected growth rate of house prices, LTV ratio, and various cost parameters, on the implied market equilibrium. When a clear sign of a partial derivative is not feasible, we use simulations to examine the impacts of the expected growth rate of house prices and mortgage interest rates on homeownership rate, equilibrium R/P ratio, and period of marginal housing tenure. We show that the order of capital cost among investors, banks, and households plays a decisive role in decisions made around housing tenure, analysis of the dynamics between rental demand and supply, and evaluation of the effectiveness of housing policies.

We also show that the homeownership rate substantially can decline as the available LTV ratio increases and when the mortgage interest rate is less than the opportunity cost of capital of investors. The magnitude of the negative impact partially depends on the market outlook: the magnitude is much smaller when the housing market outlook is optimistic, where optimistic market outlook is defined as a high expected growth rate of housing prices. We also point out in extreme conditions where there is an extremely high expected growth rate of house prices, investors may ignore the rental income and hope to purely profit from capital gain. These investors may choose to keep the housing unit vacant to keep the liquidity option open, so that they can take put the property back onto the market in a short period of time; that is, they flip the property. However, we find that the homeownership rate is insensitive to the expected growth rate of house prices in either regime; that is, a pessimistic or optimistic market outlook. A positive impact of the expected growth rate of house prices on homeownership rates is only shown when a regime switch occurs. Hence, the housing market outlook needs to be considered while evaluating or designing housing policies to increase the homeownership rate.

Affordable housing policies focus on reducing the wealth constraints of households or lowering their cost of owning to increase their incentive to become homeowners. However, many policies designed for these purposes do not always work to increase the homeownership rate. We show that the homeownership rate is determined through the competition between households and investors in bidding on a limited housing inventory in the local economy. While most home subsidy policies enhance the affordability of households so that they can purchase a house, they may also increase the profitability of the housing investments of the investors. An effective housing policy will increase the incentive of households to own *vs*. rent and also increase the local homeownership rate. However, if the same subsidy mortgage programs can be leveraged by housing investors, the greater benefits realized by the investors can outweigh the incentive to households. Thus, government subsidy funds may go to non-target receivers and result in a lower ownership rate, which negates the policy objective.

This paper develops a modeling framework to analyze the allocation of a finite housing inventory in accordance with a bidding competition between households and investors in a simple economy. The framework can be extended to incorporate tax effects into the model. That is, all cash flow items can be extended to capture various real estate related taxation issues, such as progressive income tax brackets, capital gain tax credit of the primary residence, tax deductibility of mortgage interest payments, senior and/or low income housing tax credits, etc. A follow-up paper that focuses on the tax impact on housing allocation could lead to a better understanding of the marginal impacts of various real estate taxation policies on encouraging homeownership rate or in supporting affordable rental housing policy goals. Another possible way to extend this paper is to endogenously solve for the optimal holding periods for a household. For example, Mills (1990) and Capone (1995) solved for the necessary holding period to yield a higher return than renting on the assumption that longer holding periods would always favor homeownership. An extension paper that focuses on the optimization of the rent vs. buy decision may help individual households to better plan for their housing tenure decisions.

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