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A Structural Model of a Housing Market with Friction

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This paper presents a structural model of nine equations that connect the unobserved housing service and the observed house transaction markets. Endogenous variables include two prices, supply, demand, stock of houses for sale on the market, average time on the market, stock of all houses, total vacant houses, and average house size. The search process of households for houses generates a stochastic process which results in an uncleared stock of houses on the market. The friction cost is specifically measured. The model should improve many practices in housing market research, and may be extended to other durable goods markets and beyond.

Keywords

Housing Market, Structural Model, Equilibrium, Uncleared Stock, Friction

1. Introduction

Houses are heterogeneous, durable, and generally indivisible goods that provide services for consumption by the occupying households. Their dispersed owners may act as both house sellers and buyers when they move for the sake of consumption adjustment. The total sales of used houses are of such a magnitude that they play a major role in the housing supply and reduce the relevance of the firm theory in micro- economics. Both selling and buying require a considerable amount of time. These unusual features are well known and have invited diverse approaches to model the housing market. The issues that need to be addressed are many—and seem so interrelated that makes extricating them rather difficult.

To illustrate, there is always a significant volume of houses as the **stock on market** (SOM) for sale, even when the market is apparently in equilibrium. This SOM, which violates the central tenet of neoclassic economics that a market in equilibrium is cleared, leads to a delayed **time on market** (TOM) for selling houses at an aggregate **rate of transaction** (ROT). The literature is very rich in separately explaining the housing price change with one of these three variables as the exogenous source[1]. But can we treat them as endogenous variables and related causally to each other and to price?

More significantly, the transaction price paid by a house buyer is in fact worth less to the seller after the loss as the TOM is deducted. If this loss is a function of price alone or can be separately included in the exogenous transaction cost, it should be absorbed into the supply function of the seller. Holding this assumption may explain the neglect of the issue in the past. However, what would happen if the TOM is treated as an endogenous variable? The reduced price valued by sellers due to the TOM is referred to as the effective price, or the seller's price, which is differentiated from the transaction price, or the buyer's price. Then, are there not two prices at work together to equate supply with demand?

In addition, since households purchase houses as durable goods in order to provide housing services for their own consumption, how, then, are the goods market and service equilibrium related? And how should vacant houses, which serve no one, be treated in the service equilibrium?

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[1] In the literature, housing vacancies are usually used to represent the SOM. The TOM is often referred to as vacancy duration or waiting time, and the ROT as transaction volume, turnover, matches, or the supply-demand flows when the market is in equilibrium. Linking the housing price adjustment to the SOM in terms of vacancies is a major theme in housing market research-- see footnote 7. For research on the role of the TOM in the market, see Haurin (1988), Yavas and Yang (1995), and Ong and Koh (2000); for research on the role of the ROT, see Stein (1995), Follain and Velz (1995), Hort (2000), and Caplin and Leahy (2011).

In order to address these issues in a unifying and coherent way, this paper proposes a new structural model based on a different set of constructs about the market. The intention is to form a theoretical framework that may provide fruitful and better applications. The model is composed of two related equilibriums: one of housing service aggregated from all residential households and the other of houses as goods that are being bought and sold. There are two prices in each equilibrium: one for the demand and the other for the supply. Numerous buyers and sellers enter and exit the market without restriction, all being price-takers. Yet in this competitive market, transaction friction arises from the stochastic process of house trading.

Transaction friction has been regarded as inevitable in many markets and treated in diverse ways, which still beg for a clear definition with consensus together with an unambiguous measurement[2]. In housing analyses, friction in the labor market has been often referred to as an analogy. In this paper, transaction friction is represented by the average of the TOM and the cost of friction is measured by the difference between the demand and supply prices. This is conceptually clear and simple in measurement.

The proposed model differs from all existing models with the foregoing characteristics. To repeat: the former covers both the goods and service markets, deals with them at both the individual and aggregate levels, treats the SOM, TOM, and ROT as endogenous variables related to price, etc. The model positions itself in a different category of housing structure.

In Section 2, the model is presented in four parts. The first part is the setup for the modeling with a series of definitions; the second develops two market equilibriums with nine equations; the third tackles equilibrium solutions; and the empirical estimation of the model is addressed in the fourth part. In Section 3, the contributions of the model in terms of practical applications are demonstrated. Many possible extensions from the model are outlined in Section 4. The modeling is recaptured and conclusions are made in Section 5.

[2] Petrongolo and Pissarides (2001) provide a brief history on the notion of friction in the labor market in the context of their review of "the black box of matching function". They trace the notion to Hicks, Pigou, Keynes, and its popularization in the 1970s without yielding an explicit definition associated with measurement. The early notion of friction somehow refers to voluntary, or "normal" unemployment that slows down wage adjustment. Ridder and van den Berg (2003), however, in their cross-country comparison of labor market frictions, define search friction in terms of the number of jobs offered during a spell of employment, which can be independent from wage data. Stoll (2000) reviews friction in the financial market and introduces two measurements: one by the time needed to trade an asset and the other by the price concession needed for an immediate transaction. He proposes to use the spread between bid and ask prices as the measurement of friction, and this is similar to the difference between the two prices adopted in this paper.

2. The Model

2.1 The Preliminaries

A durable good is a physical item that provides a flow of services to satisfy the user's wants. The quantitative relationship between a house as a durable good and its service flow can be simplified by properly defining their units. A house in this paper is treated, as adopted from Muth (1960) and Olsen (1969), like a physical good packed with a certain fixed volume of housing stocks, each unit of which provides one unit of housing service per time period. Due to differences in location, physical conditions, or style, houses of the same size (square footage) may contain different amounts of housing stocks. On the other hand, houses of different sizes or characteristics may contain the same volume of housing stock. The heterogeneity of houses is converted into homogenous housing service provided by a standardized housing stock.

For a community or an SOM, the total number of houses is referred to as the stock of houses. Notice the difference between the stock of houses and the housing stock defined above.

A household's demand for housing service consumption is met by its owner- - the occupier's supply that is generated from the fixed amount of his/her housing stock. The household obtains the maximum utility from an optimum mix of consumption, including that of housing service, chosen according to its preference, income and the prevailing market prices of all goods and services. Deviation from this desired mix causes disequilibrium, but this is a common occurrence and usually increases with time. After the disequilibrium reaches a threshold, it motivates a household to move and change houses in order to regain an optimum mix of consumption with housing service provided by a new volume of housing stock.

The unit price of the housing stock is not differentiated from the unit price (rental) of the housing service,[3] both of which are referred to as the housing (unit) price, denoted by p. A house k has a market price P_k as the product of the housing stock contained in the house, denoted by h_k., and the housing price p ($P_k = h_k p$). Due to the TOM, this price P_k is discounted, see Equation (9) later, to become the effective house price Π_k of the seller, which is the product of the discounted effective price of the housing unit π and the same stock $h_k.\left(\Pi_k = h_k \pi\right)$.

A community dwells on a set of houses, each of which has a different housing stock but with the same two housing prices, p and π. Houses that are vastly

[3] The unit price of a housing stock should be the capitalized value of the unit rental of the housing service. The capitalization rate is a subsumed constant in our proposed model. A more elaborate model that distinguishes a rental from price in the housing market might treat the capitalization rate as a variable in order to be connected to financial market analysis.

different in location, condition, or style are considered to belong to different communities with different housing prices. These will be treated later as different submarkets in an extension of the model.

The total stock of houses in a community, denoted by *N*, comprises two parts: one of occupied houses, denoted by *H*, each of which defines a household in the US census, and the other of vacant houses, denoted by *V*. ($N = H + V$). Houses may also be sorted as comprising another two parts: one of houses not for sale, denoted by *U*, and the other of houses as the SOM, denoted by $Z(N = U + Z)$. There are vacant houses not on the market, denoted by V_o, and, on the other hand, there are occupied houses but listed for sale, denoted by H_o. Therefore, $Z = H_0 + V - V_0$ and $U = H - H_0 + V_0$. In general, $Z \neq V$ and $U \neq H$ unless $V_o = H_o$. Note that Z, H_o, and V_o are usually unobserved.[4]

For simplicity of analysis, let both *N* and *H* have the same average housing stock per house, denoted by *h*. This implies that *V* also has the same *h*. Thus, in terms of the aggregate housing stock or housing service: $hN = hH + hV$. The average house price of the community is $P = hp$ with an average effective price of $\Pi = h\pi$.

2.2 Two Equilibriums

Although most individual owner-occupiers are not participating in the housing market, the aggregated housing service in a community is considered to be determined by the two aforementioned housing prices. Therefore, the three following equations represent the equilibrium:

$$D = d(p) \tag{1}$$

$$S = s(\pi) \tag{2}$$

$$S - hV = D \tag{3}$$

In these equations, *D* denotes the aggregate demand by households for housing services as a function ***d*** of the market price *p*, and *S* denotes the aggregate supply of housing service as a function ***s*** of the effective price of the suppliers *π*. The suppliers include both house owners and developers. The owners use the

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[4] In certain economies, the stock of vacant houses differs significantly from the SOM. Taiwan is a good example, as its 2011 housing census reports that 19.4% of houses are vacant, without further breakdown by reason. Sporadic surveys, however, show that the majority of the vacant houses are not used as seasonal houses nor for sale or rent in the market. The extremely low property tax rate, the very high cost of evicting renters in case of a default, and the long-run housing price appreciation trend in Taiwan in the past have together combined to cause a situation in which people buy houses as investment assets but keep them off the market. To distinguish SOM from vacant stock is not only essential for market analyses in such economies, but may also further facilitate data analyses in advanced economies. See Section *3.2*.3

effective price to impute rentals as the opportunity cost for providing housing service to their own household. The service available in the vacant houses serves no one, and this amount is therefore deducted from the total supply to equal the demand as shown in Equation (3).

The three equations contain six variables. To obtain a solution, as well as solve for *Z*, an SOM sub-model for the observed market of house trading is proposed below.

The SOM can be pictured as a pool of available houses with a total of *Z*. Regarding the pool, the outflow *F* represents the fall of the number of houses in the pool, and the inflow *G* represents the generation of additional houses to the pool. *F* mainly contains house purchases by households moving into the community, household formations in the community, and demolitions of existing houses. Together, they constitute the demand for houses as a function of the average house price $P = h\,p$. On the other hand, G mainly contains house sales by households leaving the community, household dissolutions in the community, and new constructions. Together, they form the supply of houses as a function of the average effective price $\Pi = h\,\pi$. The pool of SOM is in equilibrium when

$$F = f(g) \tag{4}$$

$$G = g(\Pi) \tag{5}$$

$$F = G \tag{6}$$

The task now is to postulate additional relationships that complement these three equations in order to determine not only *F, G, P*, and Π, but also the SOM and the TOM. The two prices solved are converted into their unit service prices which lead to the solution for the service sub-model of (1), (2), and (3).

Return to the SOM pool of *Z*. Although both inflow *G* and outflow *F* can be viewed as constant flows, it is better to realistically treat the individual houses in the pool as in a stochastic process of being sold. Each house sale is independent from other sales, so the process is therefore considered as Poisson with a single parameter, λ, i.e. the hazard rate. There is a very rich body of studies that separately examine certain parts of the complex house transaction process at the individual level. These studies include the decision on moving and searching on the buyer's side, the pricing strategy on the seller's side, and matching, negotiating, and settling the deals between the buyers and sellers. Only certain key aspects of these previous studies are needed to be synthesized to arrive at the hazard rate function below. The derivation of the function is anchored to the individual behaviors and presented in the Appendix.

$$\lambda = \lambda(P, Z) \tag{7}$$

In a Poisson process, the inverse of the parameter hazard rate is in fact the expected time length of the subject at risk. Therefore, in this case, the $TOM = 1/\lambda$. A house is exposed to this process once it is added into the pool of SOM. The probability of a house remaining in the pool after a time length x is $\Pr(x) = 1 - \int_0^x \lambda \exp(-\lambda t)dt = \exp(-\lambda x)$. The number of houses that are added to the pool at a time length x before and have not been sold up to this point in time is $G\Pr(x) = G\exp(-\lambda x)$. The expected total number of houses in the pool at present is hence obtained by integrating this expression over x from this point in time through all past time:

$$Z = \int_0^{\infty} G\exp(-\lambda x)dx = G/\lambda \tag{8}$$

The SOM is therefore determined.

To a representative seller who asks for a total price $P = hp$ of the house, the expected worth from the sales proceeds is this price discounted by the length of the TOM, denoted here by *w*, according to a continuous subjective discount rate *r*, and becomes $P\exp(-rw)$. Here, *w* is a random variable with a Poisson distribution that is also governed by *λ*. Therefore, the expected worth is $\int_0^{\infty} \exp(-\lambda w)P\exp(-rw)dw = \lambda P/(\lambda + r)$ .In addition to the monetary time cost represented by the seller's discount rate *r*, the seller also bears a continuous carrying cost *C* (the sum of the opportunity rent, asset depreciation, maintenance and operation costs, tax, etc.) during the selling period that should be taken into account. This cost valued at the beginning of the sale period is $C\int_0^{W} \exp(-rx)dx$. Again, *w* is a random variable with a Poisson distribution governed by *λ*. Therefore, the expected carrying cost is $\int_0^{\infty} \lambda \exp(-\lambda w)\left(C\int_0^{W} \exp(-rx)dx\right)dw = C/(\lambda + r)$. This cost should further be subtracted from the aforementioned expected worth $\lambda P/(\lambda + r)$. Thus, the seller's expected net worth from the sale, or his/her effective house price is

$$\Pi = (\lambda P - C)/(\lambda + r) \tag{9}$$

Dividing this equation by the average housing stock *h* results in the unit housing service price:

$$\pi = (\lambda - c)/(\lambda + r) \tag{9a}$$

In this equation, *c =C/P*, which represents the carrying cost rate of the house.

2.3 Equilibrium Solution
2.3.1 The Two Markets

The SOM sub-model now contains six equations (Equations (4) – (9)) to determine the six variables solved as P^o, Π^o, F^o, G^o, Z^o, and λ^o. (Hereafter, the solved equilibrium values are represented by symbols superscripted with ^{o}.)

The housing service equations of (1) – (3) are then solved as follows. A combination of the three equations yields $s(\Pi / h) - hV - d(P / h) = 0$. Given a specified supply function of ***s****(Π/ h*), a specified demand function of ***d****(P/h)*, and V (such as the observed data of vacant stock of houses), the average stock h^o is then determined after Π^o and P^o are determined in the SOM model. The factor h^o converts the two total house prices into their unit housing prices as $p^o = P^o / h^o$, $\pi^o = \Pi^o / h^o$. Then, the two unit prices respectively determine D^o in the demand function (1) and S^o in the supply function (2). The total occupied houses and the total stock of all houses at equilibrium are then determined as $H^o = D^o / h^o$ and $N^o = S^o / h^o$ respectively. Lastly, the equilibrium vacant stock is $V^o = N^o - H^o$.

The equilibrium in the aggregated housing service is therefore linked to the equilibrium of the SOM. Moreover, the model is novel in that it unveils Z^o, W^o, and h^o, which has never been attained in modeling the housing market before.

2.3.2 Sufficient Conditions for the SOM

If there were no SOM, the TOM would be nil. In such a case of a cleared market, there is no need for this model. The uncleared SOM emerges in accordance with Equation (8), as long as $G \neq 0$ and $\lambda \neq \infty$. Both conditions usually stand because there is normally a supply flow of houses into the market, and the hazard rate does not go to infinity since neither does its two components, ψ and δ in $\lambda = \psi\delta$, see the exposition that leads to Equation (7). Therefore, the existence of an uncleared SOM is not only explained as the consequence of a stochastic process, it is also assured.

2.3.3 Equilibrating

In equilibrium, $g(\pi) = G^o = f(p) = F^o$ and $\Delta Z = 0$. If there is a disturbance, say, on the supply side that causes $G > G^o$, which entails $\Delta Z > 0$, it will start the following chain reactions. Equation (9) implies $\partial \Pi / \partial \lambda > 0$, and this together with $\partial \lambda / \partial Z < 0$ in Equation (7) leads to $\Delta \Pi / \Delta Z < 0$. However, $\partial G / \partial \Pi > 0$ from the supply function (5), hence $\Delta G / \Delta Z < 0$. It reduces G and counteracts the initial disturbance $G > G^o$. With this countering, the equilibrating process continues until $G = G^o$, which restores $\Delta Z = 0$. This process applies to the situation where $G < G^o$ in the opposite direction, and also applies to the demand side when $F > F^o$ or $F < F^o$.

This process may be explored to provide the basis to relate housing price adjustments to changes in the SOM. However, the dynamic adjustment mechanism should then be explicitly specified *ad hoc*.

2.4 Empirical Estimation

The structural model which consists of Equations (1) - (9) does not need to be estimated as a whole, which would be a cumbersome task. Instead, it can be analyzed based on three sets of equations estimated separately from time series data. The first set contains Equations (4), (5), (6), and (9), and can be estimated on the basis of the postulated $\boldsymbol{f}(P)$ and $\boldsymbol{g}(\Pi)$ along with the observed time series data F^*, G^*, the average house prices P^*, and the average marketing time W^*. (Hereafter, a symbol with superscript * represents observed data for the corresponding variable.) Exogenous variables such as household income, the construction cost, etc. can all be introduced to the demand and supply functions. The second set simply contains Equations (7) and (8), and can be estimated on the basis of the postulated $\lambda(P, Z)$ along with the data of P^*, W^*, and G^*.

The third set addresses the market for housing services. It can be estimated on the basis of the postulated $\boldsymbol{d}(p)$ and $\boldsymbol{s}(\pi)$ and the following observed data: total stock of houses N^*, total occupied houses H^*, vacant houses V^*, and average transaction price P^*. The data are then converted into $\boldsymbol{S}$ $(=hN^*)$, $\boldsymbol{D}$ $(=hH^*)$, and $\boldsymbol{p}$ $(=P^*/h)$ in the model by a factor h, the average housing stock contained in all houses. The unit service supply price π is computed from p in accordance with Equation (9a). The factor h as a converting scale factor should be treated as a variable in the time series data, and therefore needs another postulated function to explain it.[5]

3. Practical Applications of Model

The proposed structural model should help to clarify many conceptual issues and answer the questions raised in the **Introduction**. Through the model, we have come to a better understanding of the housing market, including why an equilibrium market is not cleared, how the friction arises from a competitive market, why there is a price difference between the buyer and the seller, etc. The main uses of the model, however, are still those of practical applications.

[5] Section 2.3.1. states that, in equilibrium, a supply function $\boldsymbol{s}$ and a demand function $\boldsymbol{d}$ together with a given V contains a solution h^o. Here, it reminds us that h is needed to convert the data for estimating $\boldsymbol{s}$ and $\boldsymbol{d}$ in the first place.

They are achieved even if the model contains unspecified functions (1), (2), (4), (5) and (7). These applications, listed below, can all be empirically verified and assessed.

3.1 Stock-Flow Equation and Unveiled SOM

Equation (8) is rewritten as SOM = TOM × ROT. (Alternatively, in short, $R = S / T$) This can be directly reasoned out as a general stock-flow equation in equilibrium, but here, it is theoretically derived as a causal relation.[6] More significantly, since both the ROT and the inverse of the TOM are functions of price, this relationship is therefore embedded in a market process. There is an important practical use of this equation. Although the SOM is not observed, but both the ROT ($F^* = G^*$) and the average of TOM (W^*) are observed. Therefore, the SOM can be computed from these two data items in accordance with Equation (8). This computed SOM as the unveiled Z should be used in housing market research.

There is a traditional research theme which suggests that housing price adjusts according to the level of housing vacancies or changes in them. However, the empirical works seem to be inconclusive.[7] The model presented in this paper demonstrates that price and SOM, which are related to vacancies, are determined simultaneously, and that there is no direct causal relationship between the two. However, if the price adjustment is to be modeled expediently, it is the computed SOM rather than the observed vacancies that should be used, since, to repeat, vacancy data may contain houses that are not on the market.

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[6] For a stock- flow system in equilibrium, we can directly infer X = W M, where X, W, M respectively denote the size of the stock, time of each member's stay in the stock, and turnover flow (the inflow that equals the outflow). To see this, consider a college with a turnover M of 1,000 students each year (that is, 1,000 students being admitted and 1,000 students graduating each year). If each student spends four years in school, which is W, then the college has an equilibrium size X=1,000 × 4= 4,000 students. Garbriel and Nothaft (2001) apply this stock-flow relation as an identity to their study on the rental housing market.

[7] Blank and Winnick (1953) first claim, followed by Smith (1974), that there is a simple inverse relationship between housing vacancy rates and moves in housing prices or rents. Since the 1980s, there has been a surge of empirical works along this line, mainly applying regression analysis. See (among others) Rosen and Smith (1983), Wheaton and Torto (1988), and Jud and Few (1990) who confirm the relationship, and (among others) Eubank and Sirmans (1986) and Belsky and Goodman (1996) who raise doubts and criticisms. The empirical studies have advanced along pragmatic specifications of the regression equation, such as how the lagged variable should be used. See Wheaton (1990) for a review and their proposed improvements. As to whether the reference rate is stationary across cities and times or how the change in the rate should be explained, see Sivitanides (1997) for a review and his proposed improvements.

3.2 Defining and Computing Market Performance and Friction

Housing vacancy rate has commonly been regarded as excess supply, thus implying market inefficiency, and the rate has often been considered as measuring market friction. The proposed model demonstrates that both the SOM and the observed vacancy are results of the market process, and therefore may not measure the market performance and friction correctly. The market performance can now be better defined and computed by using the ratio $\Pi^o / P^o = \pi^o / p^o = (\lambda^* - c)/(\lambda^* + r)$ in accordance with Equation (9a). Here, $\lambda^*$is the inverse of the observed average marketing time *W**. Market friction that measures market inefficiency can be defined and computed as the complement to this market performance: $1 - \pi^o / p^o = (r + c)/(\lambda^* + r)$. The monetary loss to friction of an average house in the market is the difference between the market price and the effective price, or $P^o - \Pi^o$ which is $[(c + r)/(\lambda^* + r)]P^*$, and the total flow of monetary loss of the market is the product of this average house loss and ROT.

Incidentally, this endogenously determined monetary loss to friction means that the loss cannot be regarded as an exogenous cost to be absorbed into the supply function. It in turn upholds one of the critiques in the **Introduction**, and therefore justifies a departure from the traditional model of a frictionless market with clearance in equilibrium.

How substantial is this difference between the two prices in the end? It all depends on the carrying cost rate *c* and discount rate *r*. If both $c = 0$ and $r = 0$, then $\Pi = P$ in Equation (9) and $\pi = p$ in Equation (9a). Should this be the case, Equations (4), (5) and (6) will become the traditional market equilibrium model as a special case.

The difference between the two prices is certainly an empirical matter. A numerical example with assumed plausible values of *c, r*, and λ can show that this difference may be substantial indeed, especially with regard to the behavior of the builders as suppliers in a housing market.[8] As for the owner-occupiers who are involved in selling their original house and buying a new one, the variances of the three parameters should be even higher than those of the

[8] For example, if a developer has a subjective discount rate r = 0.1, and an annual carrying cost rate c = 0.1 (including the capital cost, maintenance and operation, advertising, and depreciation), an expected six months (λ=2) for selling his/her new units would result in π/p =0.90, or a loss of 10% of the sale price. By contrast, a selling time of one month (λ=12) would result in π/p = 0.98, or a loss of 2% of the price. The difference is larger than the average profit margin of most developers for projects even without considering the leverage of their equity investment, which may easily be five times or larger. It, therefore, must be the basis of the developer's planning and marketing strategies.

builders. This means that the difference could be quite significant, at least for certain households.

3.3 Estimating Speculative Holdings

In many countries, there are only aggregated statistics of vacant houses V^* without a breakdown according to reasons of vacancy. Now, after the SOM is computed from the observed ROT and TOM, we can then estimate the number of unavailable vacant houses on the market as $V_o = V^* -$ SOM. This may be further broken down into $V_o = V_1 + V_2 + V_3$, in which V_1 denotes the seasonal use, V_2 the transitional vacant houses (those already sold but waiting for new owners to move in), and V_3 the houses held off the market for speculative sales later. After assuming the portion of seasonal use and the portion of transitional vacancies, (both available in the U.S. census,) we can arrive at the amount of speculative holding V_3, which should be significant information in market research.

3.4 Hazard Rate Elasticities

Without specifying and estimating the hazard function (7), we can still make use of the equation. Combining (7) and (8) results in $Z\lambda(P,Z) - G = 0$. Totally differentiating this equation leads to $dP/P + (1/E^{\lambda}_{P} + E^{\lambda}_{Z}/E^{\lambda}_{P})dZ/Z - (1/E^{\lambda}_{P})dG/G = 0$, where E^{λ}_{P} and E^{λ}_{Z} are hazard rate elasticities with respect to price and the SOM. As the TOM = $1/\lambda$, the negatives of E^{λ}_{P} and E^{λ}_{Z} are TOM elasticities with respect to price and the SOM. With a time series of observed percentage changes in price, the turnover rate, and the computed SOM, a regression model based on this equation can be applied to estimate the two elasticities, and the postulated directions can be tested. As behavioral parameters, the two elasticities are expected to be stable and useful, and their stability can be tested with various estimations from different housing markets. However, note that the time series data should be collected at the observed equilibrium state and that the estimation is based on changes in comparative statics, not on dynamic responses.

4. Extensions

4.1 Monopolistic Competition

By substituting F for G due to the equality in Equation (6), Equation (8) can be written as $\lambda = F/Z$. The hazard rate can thus be viewed as the market demand shared by any one individual seller. A seller k of a house with a housing stock

h_k for sale is selling a distinct good since the house is indivisible. Therefore, in face of the demand λ, s/he acts as a monopolistic competitor, and chooses the optimum reservation price P_k^+ that maximizes his/her effective price $\Pi_k = (\lambda_k P_k - C_k)/(\lambda_k - r_k)$, which is extended from Equation (9) to an individual seller. Consequently, each seller has a different reservation price P_k^+ on the market.

This price can be decomposed as $P_k^+ = h_k p^o + h_k (p_k^+ - p^o)$, where p^o is the equilibrium market unit price, and $p_k^+ = P_k^+ / h_k$ is the optimum unit stock price of the seller. The component $h_k p^o$ is due to the stock size of the house, and the component $h_k (p_k^+ - p^o)$ is due to the deviation of the seller's optimum price from the market price. Hence, the observed house price spread on the market stems from two different sources. These two components can be estimated empirically with hedonic price techniques if information on the attributes of each house is available. After the estimation, the ratio p_k^+/ p^o can be computed.[9] The ratio may be further explained by the seller's discount rate r_k and carrying cost c_k in a regression investigation.

The average of p_k^+ of all sold houses may still differ from the equilibrium price p^o. This can be regarded as a source of adjustment in the market.

4.2 Submarkets

The model has been proposed for one community in which the heterogeneous houses are reduced to a homogeneous housing stock with one uniform unit price. To increase realism, the model can be extended to an urban housing market that consists of different communities, or submarkets, each of which is qualitatively so different in housing style and location that they do not necessarily share the same unit housing price.

Submarkets may simply be treated separately. Each submarket is to be modeled by the same six equations, Equations (4) – (9), and the two determined prices are then used to solve Equations (1), (2), and (3) for the corresponding service equilibrium. Demand and supply in each submarket is independent of the demand and supply in other submarkets. However, to engage in a more interactive analysis, the model can be extended to cover household movements across submarkets and make all demand and supply interdependent.

Let the submarkets be indexed by i or j. Consider the SOM pool for a submarket i, $F_i = M_{oi} + A_i + X_i$, where M_{oi} is the number of households moving into

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[9] The sale price P_k can be regressed on the housing attribute set X_k to obtain an estimate of the hedonic price coefficient $\hat{\beta}$. If we interpret $X_k\hat{\beta}$ as the $h_k p^o$ component and interpret the residual e_k as the $h_k(p_k^+ - p^o)$ component, then $p_k^+/ p^o = 1 + e_k / X_k\hat{\beta}$.

submarket i from all other submarkets, A_i is the number of households formed in the submarket, and X_i is the number of houses demolished in the submarket. M_{oi}, A_i, and X_i, are all functions of price p_i. In the same pool, on the other hand, $G_i = M_{io} + B_i + Y_i$, where M_{io} is the number of households moving out from submarket i, B_i is the number of dissolved households in the submarket, and Y_i is the number of new houses added to the submarket. M_{io}, B_i, and Y_i, are all functions of price π_i. Since $M_{oi} = \sum_j M_{ji}$ and $M_{io} = \sum_j M_{ij}$, where M_{ij} is the number of households moving from submarket i to submarket j and M_{ji} is the number of households moving from submarket j to submarket i, all the submarkets interact with each other through household moves to create and deplete house sales in origins and destinations. The proposed model can therefore be extended to become a spatial price model. That the household movement drives urban dynamics and brings about community changes is a received theme among urban students, starting probably from Grigsby (1963). Here, the interactions among submarkets are formalized.

4.3 Vacancy Transfer Chain

When a household moves from one house to another or A to B, the act can be viewed as a housing vacancy transfer from B to A in the opposite direction. Once a vacancy is created in a submarket (as a member of G in Equation (5)), the ensuing transfers form a chain through the submarkets until it is terminated (as a member in F in Equation (4)). There is a body of literature that analyzes vacancy transfers by applying the Markov chain model with a focus on the chain length as vacancy multiplier, see Hua (1989) and Emmi and Magnusson (1997). The model predicts discrete events embedded in a Markovian process without time dimension. The model proposed in this paper just may compensate the Markov chain model by providing the TOM as the vacancy's duration of stay in each submarket and, furthermore, offer market meaning to the submarket interaction of the transfers.

In short, the structural model can be integrated with an accounting system of household moves and vacancy transfers to form a spatially interactive system.

4.4 Impact Analysis and Simulation

Standard comparative static analysis can be applied to the service equilibrium in Equations (1) – (3) to study the exogenous impacts after the demand and supply functions have been estimated together with the price solutions from the SOM sub-model. However, the tool would be difficult to work with in the SOM sub-model as we need to deal with six equations; i.e., Equations (4) – (9). A simulation method may then be the option, perhaps in the form of partial comparative statics that involve the use of parameters which are separately estimated or assumed.

A computable general equilibrium method may be applicable for studying a more comprehensive multi-submarket system. To implement this, a large data base, perhaps at the scale of a metropolitan region, should be established. This would be costly but worthwhile for the potential utilities.

In order to increase realism, other factors, including the interest rate, business cycle, etc. could be incorporated into the simulation or the computable general equilibrium analysis. All dynamics in a housing market can be extended from the static equilibrium framework proposed in this paper with *ad hoc* specifications of the adjustment process.

4.5 Other Real Estate and Durable Good Markets

Section 2.3 addresses the behaviors of the agents in a market of house sales. The model should be applicable to the rental housing market as well. Potential tenants come to an apartment for rent independently in a Poisson process. They first inspect and then decide whether the apartment is worth the asked rent, based on their perceived value of the property. This process is sufficient to generate the SOM. Each landlord makes a trade-off between the desire for a higher monthly rent and a longer TOM. The situation is similar to that of a house sale. The landlord’s calculation of his/her optimal rent is slightly different from that based on Equation (9). However, as s/he will receive a rental flow rather than a lump sum of proceeds from a house sale, the tenancy length of the contract and the expected vacancy length between tenants should be included in the calculation. An adjustment of the model is needed, but this should not be difficult. Other non-housing real estate, such as offices and commercial property, should follow either the sales market model or the rental housing model adjustment.

Other resalable durable goods, such as land, used cars, jewelry, furniture, etc., which share certain basic properties of houses (namely, dispersed ownership of indivisible heterogeneous goods), should share the same market features portrayed in this paper: the TOM is long, buyers have less information than sellers and must approach sellers to inspect the goods, and sellers control the probability of a successful transaction by setting their reservation prices. The general equilibrium model presented in this paper may require modification to adapt to other specific market features for each durable good, but the basic market structural framework should remain the same.

4.6 A Behavioral Approach to Housing Bubble

The proposed model assumes that each and all agents in the market act independently and rationally. The assumption may not hold when they face an unusual situation that leads to the boom and bust of a housing market bubble. There is a long list of these new behavioral approaches to housing market bubbles. They can be explored as an extension to enrich housing market research.

5. Recapitulation and Conclusions

To model the complex housing market, two schemes have been adopted together in order to drastically simplify matters. One is to treat a physical house as packed with single-dimensioned housing stocks that yield homogeneous housing services. The other is to summarize the diverse household situations and their idiosyncratic tastes simply with their different willingness-to-pay for a house. Based on the two schemes, a rather general structural model has been built with nine equations that may capture the essential aspects of the housing market.

In the model, two equilibriums are connected: one for the housing services and the other for houses as tradable goods on the market. Each equilibrium is maintained by two prices: one for the demand and the other for the supply. It is a competitive market in which a multitude of buyers and sellers who are all price takers participate, but a stochastic process stems from the house search by individual households. The parameter of the process is endogenously determined and the process creates the SOM, TOM for selling houses, and the ROT.

Conclusions: First, in order to analyze the housing market fruitfully, the two basic tenets in microeconomics: a single price to equate supply with demand and the equilibrium clears the market, should be abandoned. Secondly, the housing market friction can be clearly defined and precisely measured, as presented in this paper. Finally, the structured model can generate many useful applications for housing market research without the specification and estimation of many functions in the model, and the merits may be judicially extended to many other markets.

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Appendix

All buyers and sellers are price takers with regard to p and π respectively. However, while a seller k knows the stock h_k contained in his/her house for sale, the buyer has to come to inspect the house in order to ascertain the housing stock.

Let a representative seller of a house with an average stock h be examined. S/he knows the prevailing housing unit price p and takes $P = hp$ as his/her willingness-to- accept (reservation) price for the house, from which s/he marks up to become his/her advertised (listing) house price.[10] A searcher-buyer, in reference to the same housing price p, searches for a house that contains stock h^+ as his/her desired optimal asset. However, s/he may not find it on the market, as each house is indivisible and we do not assume that houses in the market have a housing stock continuum. After narrowing down his/her targets in accordance with the advertised prices on the market, s/he will choose to contact a seller and go to inspect the house in order to ascertain the housing stock. Considering his/her optimal housing stock and its associated cost together with an expected additional cost if s/he continues to search, s/he will work out an amount of willingness-to-pay (bid) $P^\wedge$ for this house that would achieve the same level of utility obtainable from a house that contains h^+ with payment ph^+. If $P^\wedge \geqq P$, a deal is struck. From the seller's point of view, the searchers come to his/her house independently; hence, the house sale follows a Poisson process with a single parameter, λ, i.e. the hazard rate. Before continuing, we first need to look into this hazard rate to examine its features. The searches are generated from inside as well as outside the community. The generation rate, q per time period, can be reasonably postulated as a function $\boldsymbol{q}$ of both the aggregate housing service demand D and the availability of houses on the market measured by Z: $q = \boldsymbol{q}(D, Z)$, which, owing to $D = \boldsymbol{d}(p)$, can be simplified as $q = \boldsymbol{q}(p, Z)$, and where $\partial q / \partial p < 0$ (since $\partial q / \partial D > 0$, $d' < 0$), and $\partial q / \partial Z > 0$. The representative seller shares the total searchers with other

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[10] The assumption of a seller being a price-taker is relaxed later in Section *4.1.* to allow him/her to be a monopolistic competitor in order to determine his/her own optimum house price P_k^+.

sellers and hence faces a contact frequency $\psi = q(p, Z)/Z = \psi(p, Z)$, in which $\partial\psi/\partial p < 0$ (since $\partial\psi/\partial q > 0$ and $\partial q/\partial p < 0$) and $\partial\psi/\partial Z < 0$ (as it is assumed that the elasticity of q with respect to Z is less than one). Each searcher offers a bid $P^{\wedge}$ after inspecting the house, as mentioned. Given whatever is the distribution density of $P^{\wedge}$ over all the searchers that come to the house, the probability δ of a successful deal after contact is made depends on the cumulative probability function $\boldsymbol{\delta}$ at the seller's reservation price P: $\delta=\boldsymbol{\delta}(P)$. Multiplying this probability by the contact frequency yields the hazard rate $\lambda = \psi(p, Z)\delta(P)$. This can be simplified as:

$$\lambda = \lambda(P, Z) \tag{7}$$

where $\partial\lambda/\partial P < 0$ and $\partial\lambda/\partial Z < 0$, resultant of the derivative directions as stated above. This describes a special market operation. It is a competitive market, yet, due to the indivisibility of the goods, a stochastic process of searching and matching occurs.

Specifically, it is a Poisson process governed by a hazard rate. The main thrust is that this rate is an endogenous variable represented by Equation (7).