Recent Discoveries Related to the Use of Wittrick-Williams Algorithm to Solve Transcendental Eigenproblems

F.W. Williams & A. Watson
Department of Building and Construction, City University of Hong Kong, Hong Kong

W.P. Howson & D. Kennedy
Cardiff School of Engineering, Cardiff University, Cardiff, Wales, UK

ABSTRACT

Traditional finite element methods yield linear eigenvalue problems, the eigenvalues being undamped natural frequencies or buckling load factors. When exact elements are available, i.e. their differential equations have been solved analytically or numerically, the overall stiffness matrix is a transcendental function of the eigenparameter, e.g. it includes trigonometric and/or hyperbolic functions of expressions containing the eigenparameter. The Wittrick-Williams algorithm, which the author co-invented, must then be used to extract eigenvalues reliably.

Analogies between linear and transcendental eigenproblems have not hitherto been fully developed, even though they enable methods developed in either context to be transferred to the other and also give better understanding of existing methods. The authors have worked extensively on such analogies since January 2001 under two three year research grants. Developments completed include: the discovery of a new 'member stiffness determinant' property for all transcendental formulations which has been accepted for publication by the Royal Society, development of an inverse iteration based recursive second order transcendental eigensolver and; understanding why 'frequency squared' should be used as the transcendental eigenparameter, not the more usual 'frequency'. Exact inverse iteration substructuring methods are currently being developed to aid both transcendental and linear eigensolvers.

In mathematics, the structural engineering methods described above are for the transcendental eigenproblems with real eigenvalues which arise when large sets of differential equations are linked to form networks, where these equations may be second order Sturm-Liouville ones or of higher order. Hence the topical homogeneous tree problem of mathematics, with a transcendental eigenproblem involving over 10^{13} linked Sturm-Liouville equations, has recently been solved by using the Wittrick-Williams algorithm, and more deeply understood by using the engineer's physical understanding of the analogous structural problem.

The proposed paper and associated presentation will overview all of these developments, plus other developments to, respectively, March and July 2003.