On the Duality between Hybrid and Block Elements

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ABSTRACT

Let us consider a body decomposed into finite elements and suppose that, within each element, displacements are continuous, and no body forces are applied.

In hybrid elements:

i) the fields allowed within the element are defined through polynomial stress fields supposed to equilibrate vanishing body forces;
ii) the generalized stresses associated to each element are linear combinations of the coefficients of such polynomials;
iii) the interaction between contiguous elements is defined through compatibility conditions;
iv) the allowed fields are equilibrated, although not necessarily compatible.

In block elements:

i) the fields allowed within the element are defined through polynomial displacement fields continuous over each element;
ii) the generalized displacements associated to each element are linear combinations of the coefficients of such polynomials;
iii) the interaction between contiguous elements is defined through equilibrium conditions;
iv) the allowed fields are compatible, although not necessarily equilibrated.

Hybrid elements are such that, if displacements are prescribed on the element boundary, the field that minimizes the element total complementary energy on the linear space of the allowed (equilibrated) fields is the one belonging to that space that is nearest to the exact solution associated to such boundary displacements. Minimizing the element total complementary energy makes it possible therefore to express the generalized stresses in terms of the boundary displacements.

Block elements are such that, if tractions are prescribed on the element boundary, the field that minimizes the total potential energy on the linear space of the allowed (compatible) fields is the one belonging to that space that is nearest to the exact solution associated to such boundary tractions. Minimizing the element total potential energy makes it possible therefore to express the boundary tractions in terms of the generalized displacements.

Let us select a set of nodes on a hybrid element boundary, and suppose that a set of nodal displacements is ascribed to each node, such that the displacements along the segment between two contiguous nodes on the element boundary can be defined in terms of the nodal displacements at the ends of such segment. Then:

i) displacements along the boundary can be expressed in terms of the element nodal displacements;
ii) the approximate solution associated to the decomposition of the body into hybrid elements is obtained by equalising the generalized displacements at the coinciding nodes of adjacent elements;
iii) hybrid elements are deformable, although rigidly connected along their boundaries;
iv) deformability in a system of hybrid elements is essentially associated to the elements themselves, not to the element interfaces.

Let us select a set of nodes on a block element boundary, and suppose that the tractions are distributed on each segment between two contiguous nodes in such a way that they can be expressed in terms of generalized tractions associated to such segment. Then:

i) tractions acting on each segment can be expressed in terms of the generalized tractions;
ii) the approximate solution associated to the decomposition of the body into block elements is obtained by equilibrating the generalized tractions associated to coinciding segments of adjacent elements;
iii) block elements can themselves be rigid, although not rigidly connected along their boundaries;
iv) deformability in a system of block elements is essentially associated to the element interfaces, not necessarily to the elements.

It will be shown that such duality can be exploited so that, if the elements are small enough:

A- the subdivision of the body into hybrid elements of decreasing size provides a sequence of approximate solutions more stiff than the exact solution;
B- the subdivision of the body into block elements of decreasing size provides a sequence of approximate solutions less stiff than the exact solution.

Hybrid and block elements, used together, can thus provide lower and upper bounds to displacement values. Numerical examples will be provided.

Keywords: Hybrid elements, Block elements, duality, upper and lower bounds