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Minimum Standard of Living in the First Best Income Distribution

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Abstract

We show that the minimum standard of living can exist in a first best income distribution. The social evaluation function used in our model incorporates social value judgments of egalitarianism and individual desert. The combination of the social objective of rewarding those who deserve and the objective of egalitarianism leads to a minimum standard of living for those individuals at the bottom of income distribution. We also explore how the value judgments affect the income tax structure.

MINIMUM STANDARD OF LIVING IN THE FIRST BEST INCOME DISTRIBUTION

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1. Introduction

The minimum standard of living is usually defined as the threshold level of income for a minimum acceptable living. Such a threshold level is sometimes identified as poverty line in the study of poverty (see Sen (1976)). In practice, many social welfare programs exist over the world to ensure poor individuals to achieve a minimum standard of living. For example, China established an explicit program to implement minimum standard of living (see Chen and Barrientos (2006)). The minimum wage legislation is also aimed at allowing working people to achieve a minimum standard of living.

There are several basic questions concerning the minimum standard of living: why is there a threshold level of income called the minimum standard of living? Who decide the minimum standard of living and how is it determined? Such questions have been investigated from different aspects. To mention only a few examples, Boulding (1962, p83) describes the distribution such that "society lays a modest table at which all can sup and a high table at which the deserving can feast"; Sharif (2003) provides a behavior analysis of the minimum standard of living; Lewis and Ulph (1988) pointed out that commonly used individual utility function and social welfare function cannot give a threshold level of income, and they derive a threshold level of income by assuming non-perfectly divisible commodities. In optimal taxation literature, a threshold level of income can also be derived in a second best income distribution under incentive constraint, which is called bunching in the optimal nonlinear taxation (see Weymark (1984)).

In this paper, we provide another explanation and show that social value judgments can lead to a threshold level of income in the first best choice. Any social choice or social policy evaluation involves value judgments. (Usually we assume there is a fictional social decision maker with social value judgments). To specify explicitly value judgments, social evaluation function or social welfare function is used in economic studies. We do not restrict ourselves to use only welfarist social welfare functions, as in many literatures concerning income distributions. The Social welfare function (to distinguish it from the welfarist social welfare function, we hereafter referred it as social evaluation function) used in this paper is a function of individual incomes; it expresses the value judgments of a fictional judge or policy maker, see Creedy (2007). We try to answer the following question: what kind of social evaluation function can lead to the existence of threshold level of income? What kinds of value judgments are involved in it? Our main result is that if the social evaluation function has the following form:

$$W = (1-\theta)\sum_{i=1}^{n}\mu_i u(x_i) + \theta \min[u(x_1), u(x_2), \cdots, u(x_n)], \qquad (1)$$

where $\mu_i \geq 0$ is the weight attached to individual i with $\sum_{i=1}^{n}\mu_i = 1$, and θ is the weight assigned to the worst off individual with $0 \leq \theta \leq 1$, then in the optimal income distribution with lump sum transfer, there exists a threshold level of income. In the optimum, all individuals with weight μ less than a certain level receive the same level of income, while the incomes of other individuals increase strictly with their weights in the social evaluation function.

The social evaluation function looks like a combination of weighted utilitarianism and Rawlsian egalitarianism. We interpret the curvature of function u as the inequality aversion of the fictional social decision maker. The weights μ_i attached to individuals and the weight θ to the worst off individuals play important role in our model. The weight μ_i can represent the fictional social decision maker's value judgment on the desert (or power) of individual i. An individual who is considered to deserve more will get more income than an individual who is considered to deserve less. The weight μ_i provides a way to describe the important idea of individual desert which is largely ignored in economic analysis. The weight θ represents the value judgments of egalitarianism, which tends to give each individual equal income. The compromise of such opposite effects leads to a threshold level of income, which is the minimum standard of living.

A natural question to ask is how the weights μ_i and θ and the function u are determined. These parameters represent the decision maker's actual value judgments and can be derived from the revealed preferences. Therefore, we are interested at value judgments that are used actually in decision making rather than value judgments that should be used.

We apply the model to income taxation. We consider a special social value judgment of desert that is based on the proportional principle by Aristotle, so that the individual weight is proportional to the pretax income. We obtain minimum standard of living for the poor, and we also obtain interesting result on the regressivity or progressivity of taxation based on social value judgments, without appealing to the equal sacrifice principle. Income is progressive or regressive depends on whether the

Arrow Pratt coefficient of function u is greater or less than 1. This implies that progressive taxation is the outcome of high social inequality aversion.

The rest of this paper is organized as follows. Section 2 derives the optimal income distribution for social evaluation function (1). Section 3 discusses why it is appropriate to use social evaluation function (1). Section 4 discusses the implication in income taxation. Section 5 is the conclusion.

2. Deriving minimum standard of living in the first best income distribution

Before we look at whether it is reasonable to use the social evaluation function (1), we first derive the optimal income distribution of a fixed total income w among n individuals (or any social units such as households) that maximizes the given social evaluation function (1). The function u is concave and lump sum transfer is possible. A feasible income distribution is a vector $x=(x_1,\cdots,x_n)$ such that $x_i \geq 0$ and $\sum x_i = w$. (Without confusion, we also use $x=(x_1,\cdots,x_n)$ to denote the optimal income distribution in the following.) We rank individuals according to their weights so that $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_n$. To remove some tedious mathematical notations, we first assume that $\mu_1 < \mu_2 < \cdots < \mu_n$ to get our main results. For the general case with $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_n$, we only need to regroup the individuals and use similar proof. We will come back briefly to this issue later. Theorem 1 in this section is our main result. In Proposition 1 we provide an easy way to solve the problem of optimal distribution.

To prove our main result, we first prove two lemmas. Lemma 1 shows that individual 1, who is the individual with the lowest weight, has the lowest income. Lemma 2 shows that the ranking of individual incomes in the optimum is the same as the ranking of individual weights. The proofs use the Kuhn-Tucker theorem, and we put them in the appendix.

Lemma 1: In the optimal income distribution, individual 1 has the lowest income, i.e., $x_k \geq x_1$ for all $k > 1$.

When $\theta = 0$ and u is strictly concave, the first order condition $u'(x_i)/u'(x_j) = \mu_j / \mu_i$ implies $x_1 < x_2 < .... < x_n$. An individual with strictly higher weight should have strictly higher level of income. This is no longer true with egalitarianism consideration (i.e., $\theta > 0$). Two individuals with different weights may end up with the same income. The lemma implies that no matter how egalitarian a social decision maker is, the individual with the lowest weight cannot be strictly better off than any other individual.

Lemma 2: In the optimal income distribution, we have $x_1 \leq x_2 \leq .... \leq x_n$.

This lemma implies that even with egalitarianism consideration, individual with higher weight should not end up with strictly less income than those with less weight. The ranking of individual income should respect the ranking of individual in the social value judgment.

Given the above two lemma, it is easy to show our main result: (see appendix for proof).

Theorem 1: To maximize social evaluation function (1) with $\mu_1 < \mu_2 < \cdots < \mu_n$, there exists a unique value k^* such that in the optimal income distribution, $x_1 = x_2 = \cdots = x_{k^*} < x_{k^*+1} < \cdots < x_n$. The value of k^* is determined as the smallest number $l \in \{1,...,n\}$ such that $\theta < (1-\theta)\sum_{j=1}^{l} \mu_{l+1} - \mu_j$.

Therefore, in the optimal income distribution there is a threshold level of individual weight so that individuals with the weight less than the threshold level have the same income, and other individuals have income strictly increasing with their weights. The threshold level of income in the optimum for the poorest k^* individuals, denoted by $\bar{x}$, can be considered as the minimum standard of living. If we assume that the weight of an individual is proportional to his pre-redistribution income (we will discuss this assumption in the next section), individuals from 1 to k^* can be categorized as the poor in the study of poverty and $\bar{x}$ can be explained as the poverty line.

It is interesting to note that the threshold value k^* is independent of the form of function u and the level of total income w. The intuition behind this fact is that the ranking of individual incomes is determined by the ranking of the individual weights, independent of the total wealth and the curvature of function u. When total income and the social attitude towards inequality change, the threshold value k^* will not change, but the level of minimum standard of living will change.

To find the threshold value k^*, we can follow the following steps: if $\theta < (1-\theta)(\mu_2 - \mu_1)$, then $k^* = 1$. Otherwise, if $\theta \geq (1-\theta)(\mu_2 - \mu_1)$ but $\theta < (1-\theta)(\mu_3 - \mu_1 + \mu_3 - \mu_2)$, then $k^* = 2$, and so on. After calculating the threshold value k^*, the optimal income distribution can be calculated using the following proposition:

Proposition 1. Once the threshold level k^* is determined, the optimal income distribution maximizing (1) can be calculated by maximizing the social evaluation function $W = \sum_1^n \bar{\mu}_i u(x_i)$, where $\bar{\mu}_1 = \cdots = \bar{\mu}_{k^*} = \dfrac{\theta + (1-\theta)\sum_1^{k^*} \mu_i}{k^*}$ and $\bar{\mu}_j = (1-\theta)\mu_j$ for $j > k^*$.

Proof: see appendix. ■

We can see from proposition 1 that egalitarianism consideration takes effects through changing the individual weights. A weight θ on egalitarianism reduces the high individual weights proportionally by a factor of $1-\theta$, and make the rest of the low weights to an equal level. Thus, the optimal income distribution with egalitarianism consideration can be solved through a standard problem of optimal income distribution (without egalitarianism consideration). This greatly simplifies the problem.

Since k^* is the smallest value of l such that $\theta < (1-\theta)\sum_{j=1}^{l}(\mu_{l+1}-\mu_j)$ and the ratio $\theta / 1-\theta$ is an increasing function of $\theta \in [0,1)$, it is clear from Theorem 1 that as θ increases, the number k^* of individuals with the minimum standard of living increases. We can also prove the following proposition:

Proposition 2. Holding other things fixed, an increase in the weight on egalitarianism will increase the threshold level k^* and increase the level of the minimum standard of living.

Proof: We use (8) in the appendix and use the fact that as θ increases, $\overline{\mu}_i / \overline{\mu}_1$ for $i > 1$ are decreasing function of θ. The detailed proof is omitted. ■

Thus, a more egalitarian society will set a higher minimum standard of living. If we consider the threshold level of income as the poverty line, then a more egalitarian society will set a higher poverty line and categorize more individual as in poverty.

We have proved our main results for $\mu_1 < \mu_2 < \cdots < \mu_n$. For general cases with $\mu_1 \le \mu_2 \le \cdots \le \mu_n$, we can group individuals according to their weights in the social evaluation function. Individuals with the same weight should have the same income (if not, equalization of income among them will increase the value of the social evaluation function W because u is strictly concave). Suppose there are j

different groups. There are n_i individuals in group i, each with weight μ_i and with the same income x_i, and $n_1+\cdots+n_j=n$, $\mu_1<\mu_2<\cdots<\mu_j$. The proof for the general case is very similar to the case already discussed. For example, the Lagrangian in lemma 1 becomes:

$$\begin{aligned}L&=(1-\theta)n_1\mu_1u(x_1)+\cdots+[(1-\theta)n_k\mu_k+\theta]u(x_k)+\cdots+(1-\theta)n_ju(x_j)\\&-\gamma(\sum\nolimits_1^j n_ix_i-w)-\sum\nolimits_{i\neq k}\lambda_i(x_k-x_i)\end{aligned}$$

We omit the proof here. For the general case, there is still a threshold value k^* such that $x_1=x_2=\cdots=x_{k^*}<x_{k^*+1}<\cdots<x_j$. Individuals in the first k^* groups (the poor) all have the same income, which is the minimum standard of living. Other individuals' income is strictly increasing with their weights. The threshold value k^* is the smallest number $l\in\{1,\cdots,n\}$ satisfying $\theta+(1-\theta)\sum_{i=1}^{l}n_i\mu_i<(1-\theta)(\sum_{i=1}^{l}n_i)\mu_{l+1}$, or equivalently, $\theta<(1-\theta)\sum_{i=1}^{l}n_i(\mu_{l+1}-\mu_i)$. To find the optimal income distribution, the weight in proposition 1 is calculated by $\bar{\mu}_1=\cdots=\bar{\mu}_{k^*}=\dfrac{\theta+(1-\theta)\sum_1^{k^*}n_i\mu_i}{\sum_1^{k^*}n_i}$.

3. Is the social evaluation function (1) reasonable?

We have shown that if a fictional social decision maker has a social evaluation functions with form (1), he will set a threshold level of income as the minimum standard of living, and he will give higher income to other individuals with higher weights. Now we need to ask whether it is reasonable to use such a form of social evaluation function in social decision making. Are there justifications to use such evaluation function to describe social decision making? Can it have relevant empirical significance?

In social decision makings (or more generally, any collective decision making) different decision makers can make different choices under similar context. This implies that different social evaluation functions are used, and the social value judgments are different. As a general form of social evaluation function, the social evaluation function (1) includes two forms of widely used evaluation function as its special cases: a weighted additive social evaluation function $W = \sum_{i=1}^{n} \mu_i u(x_i)$ and a Rawlsian maximin social evaluation function $W = \min[u(x_1), u(x_2), \cdots, u(x_n)]$.

We first look at the weighted additive part of the social evaluation function $W = \sum_{i=1}^{n} \mu_i u(x_i)$. It is well known that the Arrow-Pratt coefficient $-u''/u'$ of the function u represents the decision maker's inequality aversion. In literature of weighted additive social welfare function, the weights usually represent the population density while each individual still has the same weight. In our social evaluation function each single individual has different weight. In the optimal income distribution, the first order condition $u'(x_i)/u'(x_j) = \mu_j / \mu_i$ implies that $x_i > x_j$ for $\mu_i > \mu_j$. Therefore, an individual with higher weight will get more income. The weights attached to individuals can represent the fictional decision maker's value judgment on individual desert: an individual with high weight implies he deserves more. In other situation, we may consider the weight as individual power or individual need.

In most literature all individuals are assumed to be treated equally. Inequality is not the first best; it is allowed only in the second best when incentive constraints are binding. However, in our model income inequality is socially optimal as the first best because of the value judgment of individual desert. The idea of individual desert is popular in the discussion of income distribution in sociology and philosophy, but it is largely ignored in economics. In Aristotle's Nicomachean Ethics, inequality is optimal: justice is the proportional; and award should be according to merit.

Then, in a social evaluation function, how the individual weights (value judgments of desert) are determined? The weights are parameters of the decision maker's preferences. The process of preference formation is complicated, but we can consider the process of preference formation as a black box and derive the parameters from revealed social preferences. Therefore, we do not need to impose exogenously what is the correct value judgments that should be used in social decision making.

Social decision making are context sensitive, and so are the value judgments of desert. For example, for income taxation the weights may be positively related to their pretax incomes. This reflects the value judgments that those who create more wealth deserve to get more income. However, to distribute a certain resource (unrelated to effort or contribution) to individuals in order to reduce income inequality, the weights are very likely inversely related to the individual pre-distribution income. Those who are poor deserve to get more. If decision making is determined by political power, the weights may be positively related to the relative political power of individuals.

The maxmin part of the social evaluation function $W = \min[u(x_1), u(x_2), \cdots, u(x_n)]$ represents the decision maker's value judgment of egalitarianism. Egalitarianism represented by the weight assigned to the worst off individual is different from inequality aversion represented by the concavity of u. Inequality aversion is equivalent to decreasing marginal social value from individual income. The difference between the two can be seen from the optimal income distributions of a total wealth under $W = \sum_{i=1}^{n} \mu_i u(x_i)$ and under $W = \min[u(x_1), u(x_2), \cdots, u(x_n)]$. To maximize $W = \sum_{i=1}^{n} \mu_i u(x_i)$, if u is concave (i.e., inequality aversion) individual incomes are equal only if all individuals have the same weights. (If u is strictly convex, income will be concentrated to one individual). However, to maximize $W = \min[u(x_1), u(x_2), \cdots, u(x_n)]$, individual incomes will be equal for any function u. Compared with inequality aversion, egalitarianism has much stronger effect to make income equal.

Therefore, social evaluation function in (1) actually incorporates three different value judgments of the decision maker: inequality aversion, individual desert and egalitarianism. This is very appealing for a social evaluation function; it can leads to new theoretical results and at the same time the problem is tractable. Questions may be raised concerning whether such social evaluation functions have reasonable theoretical foundation. In Zheng and Anwar (2005) we derive social welfare function $W = \sum_{i=1}^{n} \mu_i u(x_i)$ from the consistency of legal decision making by applying the Savage approach, and use the function to discuss the value judgments in legal decisions. In the paper, we assumed that social decision making is consistent in the sense of Savage axioms. In original Savage approach, the decision maker has preference over acts. Each act is a vector $(x_1, \cdots, x_n)$, which is the overcome of different states of the world if an action is taken. It is very natural to interpret the vector as an income distribution among n individuals, and the Savage axioms can be explained as the consistency over the choices of income distribution (For detailed discussion, see Zheng and Anwar (2006)). With this reinterpretation, if the decision maker's preference is consistent (in the sense of Savage axioms) over income distribution, a cardinal social welfare function $W = \sum_{i=1}^{n} \mu_i u(x_i)$ is uniquely determined up to an affine transformation of u. Therefore, from revealed preference satisfying Savage axioms, we can derive uniquely the decision maker's subjective value judgments of inequality aversion and value judgment of desert (or individual power as used in Zheng and Anwar (2006)).

The additive evaluation function (and correspondingly, the independent axiom in Savage axioms) is very restrictive for social decision making as it excludes any form of egalitarianism consideration. We can use a less restrictive form of consistency axioms, which is referred to as the Savage Theorem without independent axiom, to get a non additive function W [1]; see Gilboa (1993). The social evaluation function (1) is a special case of the function W derived from the consistency of social decision making without the independence axiom.

It is controversy whether there is a social entity that can make consistent social decision making, but we can avoid this controversy by assuming the existence of a fictional decision maker. Therefore, the social evaluation function in form (1) is not arbitrarily imposed exogenously; it can describe a large class of social decision makings satisfying reasonably consistency conditions. [2]

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[1] Actually, all such function can be written as $W = \sum_{A \subset \Omega} \mu_{\mathrm{A}} \min_{x \in A} u(x)$, where the summation is over all subset of Ω, with Ω denotes the set of all individuals, see Gilboa and Schmeilder (1994). Therefore, it assigns weights to each individual, and it also assigns weights to all possible coalition of individuals. Social evaluation function (1) only assigns weights to single individuals and to the whole group.

[2] In a different setting in lab experiments about income distribution by individuals, it is claimed that the decision making involves the combination of inequality aversion, efficiency, and maximin consideration, see Engelmann and Strobel (2004).

4. Implication on income redistribution and taxation

In section 2, we assume that the total income and the social evaluation function are given exogenously. Now we consider a more realistic case about how to redistribute earned income among individuals. We make two assumptions: 1. The pre-redsitribution earned incomes $x^0 = (x_1^0, \cdots, x_n^0)$ are given (we ignore the incentive effect in this paper), and $w^0 = x_1^0 + \cdots + x_n^0$. The earned income of individual i is x_i^0. 2. The function u and the weight θ to egalitarianism are given, but the weight attached to individual i in the social evaluation is proportional to his earned income:

$$\mu_i = x_i^0 / w^0 .$$

Thus the fictional social decision maker's value judgments of inequality aversion and egalitarianism are given, but individual desert is proportional to his contribution, which corresponds to the proportional principle in Aristotle's ethics. (There is no reason to assume that value judgments of inequality aversion and egalitarianism cannot change, but for income distribution with given earned income, it is reasonable to assume that they will not change with the pretax income. Also, the proportional principle may be substitute by a form to allow the weight to be a general increasing function of earned income.)

Suppose the value judgment of inequality aversion takes the form of:

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

for $\gamma \neq 1$, and $u(x) = \ln x$ for $\gamma = 1$. Here γ is a constant degree of relative inequality aversion of the fictional decision maker.

We first consider the case when there is no egalitarianism consideration (i.e., $\theta = 0$).

For $u = \ln(x)$ and $\mu_i = x_i^0 / w^0$, the first order condition $x_i / x_j = \mu_i / \mu_j$

implies that the final income of an individual will be proportional to his weight, thus $x_i = \mu_i w^0 = x_i^0$. Each individual keeps his own pretax income and no redistribution or taxation is needed in the optimum.

When $\gamma \neq 1$ and $\theta = 0$, the first order condition becomes $(x_i / x_j)^\gamma = \mu_i / \mu_j$. In terms of earned income, we get $x_i / x_j = (\mu_i / \mu_j)^{1/\gamma} = (x_i^0 / x_j^0)^{1/\gamma}$, or

$$\frac{x_i / x_i^0}{x_j / x_j^0} = (x_i^0 / x_j^0)^{1/\gamma - 1}.$$

If $\gamma > 1$, then $x_j / x_j^0 < x_i / x_i^0$ if $x_j^0 > x_i^0$. Individual with higher earned income (the rich) get smalled proportion of their earned incomes, and individuals with low earned income get bigger proportional of their incomes. This is a form of progressive taxation in the sense that the average tax rate increases with income. As we only consider income redistribution, some individuals must be subsidized if others are taxed. In this case, the rich is taxed to subsidize the poor. Therefore, we get progressive taxation when the relative inequality aversion coefficient γ is greater than 1.

Similarly, for $\gamma < 1$, we have $x_j / x_j^0 > x_i / x_i^0$ if $x_j^0 > x_i^0$. Thus we get a regressive taxation. Tax rate is higher for poor individuals. Thus, the rich will be subsidized and the poor will be taxed even though the decision maker is inequality averse. This happens because the effect of individual desert dominates the effect of inequality aversion.

Without egalitarianism consideration, if individual desert is proportional to pretax income, taxation will be progressive if the decision maker is highly inequality averse (i.e., $\gamma > 1$); taxation will be regressive if the inequality aversion of the decision maker is not so strong (i.e., $\gamma < 1$); and there will be no redistribution if

$\gamma = 1$. This result, though simple in our model, provide another insight about progressivity or regressivity of taxation by using the value judgments of desert and inequality aversion. In previous literature, the common way to derive the progressivity of taxation is though the use of the equal sacrifice principle, see Samuelson (1947).

If in addition the society has egalitarianism consideration, there will be a minimum standard of living for the k^* individuals with low earned income. The exact form of taxation can be derived through two steps by Proposition 1 in section two. In the first stage, earned incomes are redistributed by levy a fixed tax rate θ to individuals with high earned income and give other k^* individuals the same income. Then in the second stage, the incomes achieved at the end of the first stage are going to be redistributed as discussed above, when there is no egalitarianism consideration.

When $\gamma = 1$, there is no redistribution at the second stage. We get an example of minimum standard of living, combined with a linear tax rate[3] (at least for individual with higher earned income). However, this scheme is different from that in the basic income/flat tax proposal as discussed in Atkinson (1995). In the basic income proposal, all individuals, poor or rich, will be given a fixed basic income and all individuals facing the same tax rate.

Even without considering the incentive effects, such results are not trial for the existing literature. It shows the important roles that value judgments such as individual desert, inequality aversion and egalitarianism can play in social decision making.

[3] Without considering the incentive effect, it is possible that some individuals' pretax incomes are higher than the minimum standard of living but their post-tax income is at the minimum standard of living. However, they are taxed at a rate lower than θ. This will not happen when incentive is considered, since individuals who get the minimum standard of living will not work anymore.

Our model suggests a subjective view of the minimum standard of living (and poverty line). The level of minimum standard of living, and criterion for who should receive income at the minimum standard of living, depends on the social value judgments. A more egalitarianism social decision maker would put more individuals in the category of poor and would set a higher minimum standard of living. From theorem 1, the inequality aversion has no effect on determining who should be categorized as poor, though it has effect on the level of the minimum standard of living.

5. Conclusions

This paper shows that the minimum standard of living is required in the social optimum if the social decision maker has three combined value judgments: egalitarianism, individual desert and inequality aversion. For a social decision maker, incomes should be unequal to reward those who deserve more, but egalitarianism requires all individuals have the same income. The compromise of these two leads to a minimum standard of living. We believe our model can provide a useful analytic tool to study the value judgments, especially the value judgment of individual desert and egalitarianism in the economic analysis.

For further study we need to consider the incentive of labor supply. If we consider ex-post redistribution in which government redistribute earned incomes after earned incomes are realized, it is easy to see that those who get minimum standard of living will choose not to work. But if we use the framework of optimal nonlinear taxation in which government first designs a tax scheme to let individuals to self-select their labor supply, the problem will be more complicated to solve. We will discuss these issues in a separate paper.

Appendix:

Proof of Lemma 1: We prove the lemma by contradiction. Suppose that individual $k \neq 1$ has the lowest income and $x_k < x_1$. Then social welfare function (1) becomes:

$$W = (1-\theta)\mu_1 u(x_1) + \cdots + [(1-\theta)\mu_k + \theta]u(x_k) + \cdots + (1-\theta)\mu_n u(x_n) .$$

The constraints for the maximization are $x_k \leq x_i$ for all $i \neq k$ and $x_k < x_1$. The associated Lagrangian for this constrained optimization is:

$$L = (1-\theta)\mu_1 u(x_1) + \cdots + [(1-\theta)\mu_k + \theta]u(x_k) + \cdots + (1-\theta)\mu_n u(x_n)$$
$$-\gamma(\textstyle\sum_{i=1}^{n} x_i - w) - \displaystyle\sum_{i \neq k} \lambda_i (x_k - x_i)$$

where γ, λ_i's are multipliers. The corresponding Kuhn-Tucker conditions include:

$$(1-\theta)\mu_i u'(x_i) - \gamma + \lambda_i = 0 \text{, for } i \neq k , \tag{2}$$

$$[(1-\theta)\mu_k + \theta]u'(x_k) - \gamma - \sum_{i \neq k} \lambda_i = 0 , \tag{3}$$

$$\lambda_i (x_k - x_i) = 0 \text{ with } \lambda_i \geq 0 \text{ and } x_k \leq x_i \text{ for } i \neq k . \tag{4}$$

From $x_k < x_1$ (by assumption), (4) implies $\lambda_1 = 0$. Using $\lambda_1 = 0$ in (2) we have $(1-\theta)\mu_i u'(x_i) = \gamma - \lambda_i \leq (1-\theta)\mu_1 u'(x_1)$, for all $i \neq 1$, k. As $\mu_1 < \mu_i$, it follows that $u'(x_i) < u'(x_1)$. By the concavity of u, $x_i > x_1 > x_k$ for all $i \neq 1, k$. From (4), then $\lambda_i = 0$ for all $i \neq k$. Then (3) simplifies to $[(1-\theta)\mu_k + \theta]u'(x_k) = \gamma = (1-\theta)\mu_1 u'(x_1)$. From $\mu_1 < \mu_k$, then $(1-\theta)\mu_k + \theta > (1-\theta)\mu_1$, and so $u'(x_k) < u'(x_1)$. By the concavity of u, $x_k > x_1$, which is in contradiction with the initial assumption $x_k < x_1$. ■

Proof of Lemma 2: Since the first individual has the lowest income, (1) becomes $W = [(1-\theta)\mu_1 + \theta]u(x_1) + (1-\theta)\mu_2 u(x_2) + \cdots + (1-\theta)\mu_n u(x_n)$. The

Kuhn-Tucker conditions with resource constraint $\sum_{i=1}^{n} x_i = w$ and $x_1 \le x_i$ for all $i \ne 1$ include:

$$(1-\theta)\mu_i u'(x_i) - \gamma + \lambda_i = 0 \text{, for } i \ne 1, \qquad (5)$$

$$[(1-\theta)\mu_1 + \theta]u'(x_1) - \gamma - \sum_{i \ne 1} \lambda_i = 0, \qquad (6)$$

$$\lambda_i (x_1 - x_i) = 0 \text{ with } \lambda_i \ge 0 \text{ and } x_1 \le x_i \text{ for } i \ne 1. \qquad (7)$$

We show, by contradiction, that there exists no $k > 1$ such that $x_{k+1} < x_k$. Suppose $x_{k+1} < x_k$ for some $k > 1$. As individual 1 is the poorest, $x_{k+1} < x_k$ implies that $x_1 < x_k$ and (7) implies $\lambda_k = 0$. Using $\lambda_k = 0$ in (5) we have $(1-\theta)\mu_{k+1} u'(x_{k+1}) = \gamma - \lambda_{k+1} \le (1-\theta)\mu_k u'(x_k) = \gamma$. Because $\mu_k < \mu_{k+1}$, we obtain $u'(x_{k+1}) < u'(x_k)$. The concavity of u implies $x_{k+1} > x_k$, which contradicts the initial assumption $x_{k+1} < x_k$. ■

Proof of Theorem 1: First we prove that if $x_l < x_{l+1}$, then $x_l < x_{l+1} < \cdots < x_n$. (This is equivalent to that $x_l = x_{l+1}$ implies $x_1 = x_2 = \cdots = x_{l+1}$). Suppose that $x_l < x_{l+1}$, then one must have $x_1 < x_{l+1}$ and by condition (5) to (7), $\lambda_{l+1} = 0$ $(1-\theta)\mu_{l+2} u'(x_{l+2}) = \gamma - \lambda_{l+2} \le (1-\theta)\mu_{l+1} u'(x_{l+1}) = \gamma$. Because $\mu_{l+1} < \mu_{l+2}$ and by the concavity of u, we obtain $x_{l+1} < x_{l+2}$. Recursively, we get $x_{l+2} < x_{l+3}, \cdots$, and $x_{n-1} < x_n$. Therefore there must exists a unique threshold value k, which is the smallest number of l satisfying $x_l < x_{l+1}$, such that $x_1 = x_2 = \cdots = x_k < x_{k+1} < \cdots < x_n$.

Next, we investigate how the threshold value k can be determined. We prove the result by two steps. Step 1: for an l, if $\theta \ge (1-\theta)\sum_{j=1}^{l} (\mu_{l+1} - \mu_j)$, then

$x_l = x_{l+1}$. Step 2: for an l, if $\theta < (1-\theta)\sum_{j=1}^{l}(\mu_{l+1} - \mu_j)$, then $x_l < x_{l+1}$. Combining step 1 and 2, then k is the smallest number l satisfying $\theta < (1-\theta)\sum_{j=1}^{l}(\mu_{l+1} - \mu_j)$

Step 1: If $x_l < x_{l+1}$, then $x_1 < x_{j+1}$ for all $j > l$, therefore by (9), $\lambda_{l+1} = \cdots = \lambda_n = 0$. The conditions (6) and (5) imply that for individual 1, $[\theta + (1-\theta)\mu_1]u'(x_1) - \gamma - \lambda_2 - \cdots - \lambda_l = 0$ and for individual $j = 2,....,l$, $(1-\theta)\mu_j u'(x_j) - \gamma - \lambda_j = 0$. Adding the equation for individual 1 to l, we get:

$$
\begin{aligned}
&[\theta + (1-\theta)(\sum_{j=1}^{l}\mu_j)]u'(x_l) \le [\theta + (1-\theta)\mu_1]u'(x_1) + \sum_{j=2}^{l}(1-\theta)\mu_j u'(x_j) \\
&= l\gamma = l(1-\theta)\mu_{l+1}u'(x_{l+1})
\end{aligned}
$$

(The inequality is obtained from the condition $x_1 \le x_2 \le \cdots \le x_n$ and from the concavity of u. The first equality is obtained by adding the equations for individual 1 to k and the last equality is by (5) and the fact that $\lambda_{l+1} = 0$.)

As $x_l < x_{l+1}$, in order the above relation to be true, one must have $\theta + (1-\theta)\sum_{j=1}^{l}\mu_j < l(1-\theta)\mu_{l+1}$. Equivalently, if $\theta \ge (1-\theta)\sum_{j=}^{l}(\mu_{l+1} - \mu_j)$, then one cannot have $x_l < x_{l+1}$ and it must be true that $x_l = x_{l+1}$ (as we have $x_1 \le x_2 \le .... \le x_n$ from lemma 2).

Step 2: If $x_l = x_{l+1}$ then $x_1 = ... = x_{l+1}$, (as is known from the beginning of this proof). From (6) and (5), it follows that for individual 1, $[\theta + (1-\theta)\mu_1]u'(x_1) - \gamma - \lambda_2 - \cdots - \lambda_l - \lambda_{l+1} - \cdots - \lambda_n = 0$ and for individual $j = 2,3,\cdots,l$, $(1-\theta)\mu_j u'(x_j) - \gamma - \lambda_j = 0$. Adding the equation for individual 1 to l and taking into account that $x_1 = \cdots = x_l$, we get

$[\theta+(1-\theta)\sum_{j=1}^{l}\mu_j]u'(x_l)=l\gamma+\sum_{i\geq l+1}\lambda_i$. For individual $l+1$, (5) implies $(1-\theta)\mu_{l+1}u'(x_{l+1})=\gamma-\lambda_{l+1}$. Therefore, we have:

$$[\theta+(1-\theta)(\sum_{j=1}^{l}\mu_j)]u'(x_l)\geq l\gamma\geq l(1-\theta)\mu_{l+1}u'(x_{l+1})$$
.

As $x_l=x_{l+1}$, the above relation implies $\theta+(1-\theta)(\sum_{j=1}^{l}\mu_j)\geq l(1-\theta)\mu_{l+1}$. Equivalently, if $\theta+(1-\theta)(\sum_{j=1}^{l}\mu_j)<l(1-\theta)\mu_{l+1}$ (or $\theta<(1-\theta)\sum_{j=}^{l}(\mu_{l+1}-\mu_j)$), then it is impossible to have $x_l=x_{l+1}$ and we must have $x_l<x_{l+1}$ (as we have $x_1\leq x_2\leq....\leq x_n$ from lemma 2). ■

Proof of Proposition 1: Once k^* is determined, condition (5) implies $(1-\theta)\mu_{k+1}u'(x_{k+1})=\cdots=(1-\theta)\mu_n u'(x_n)=\gamma$. For individual $i=2$ to k^*, (5) implies that $(1-\theta)\mu_i u'(x_i)-\gamma+\lambda_i=0$. For individual 1, from (6), $[(1-\theta)\mu_1+\theta]u'(x_1)-\gamma-\sum_{i\neq 1}\lambda_i=0$. Adding the equations for individual 1 to individual k^*, we get $[\theta+(1-\theta)\sum_{i=1}^{k^*}\mu_i]u'(\bar{x})=k^*\gamma$. Therefore, the optimal income distribution is determined by much simpler conditions:

$$[\theta+(1-\theta)\sum_{i=1}^{k^*}\mu_i]u'(\bar{x})=k^*(1-\theta)\mu_{k^*+1}u'(x_{k^*+1})=\cdots=k^*(1-\theta)\mu_n u'(x_n) \qquad (8)$$

with the constraint $k^*\bar{x}+x_{k^*+1}+\cdots+x_n=w$. Then we can easily prove the proposition. ■

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